THE OPTIMAL CURRENCY DENOMINATION STRUCTURE: A CASE OF THAILAND

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ABSTRACT

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Payment systems nowadays are encouraged to move toward electronic payments (e-payments), which are expected to be more efficient, with low cost and high convenience for making payments. However, cash cannot be perfectly substituted by e-payment and is still needed in society. Besides the promotion of policy towards e-payment, cash denomination should be considered and restructured to be compatible with the real demand, especially when there are various changes in circumstances such as e-payment usage, price level, and social preferences.

This research examines optimal currency denomination structure from the cost and cash payment efficiency perspectives, focusing on banknotes. The study proposes conceptual methodology, together with empirical study and numerical analysis for Thailand. The main components used to set up optimization problems are: i) banknote demand by denomination derived from characteristic model; ii) cash payment efficiency based on the principle of least effort and Cramer's model; iii) the D-Metric model for controlling the boundary of currency denomination structure; and iv) the projection of relevant exogenous factors. Considering all components together, numerical solutions would be obtained from the simulation-optimization technique.

The analyses were done from static and dynamic views and with different purposes. The static analysis was for examining whether the current denomination structure is optimal, while the dynamic analysis was applied to find the optimal denomination structure with multi-period determination. The denomination restructuring is then forwardly planned for Thailand. The results show that the current banknote denomination structure in Thailand is not optimal, according to the present approach, and the 20-year anticipated plan will eliminate 50-baht banknotes. Five years from now (2016), 200-baht banknotes should be introduced into circulation. In 2021, 50-baht banknotes will re-enter the denomination system, together with a new 2000-baht banknotes. At the same time, 20-baht banknotes should be replaced by 20-baht coins. According to this optimal plan, cost and cash payment efficiency would improve.

This study could be used as a guideline for the Bank of Thailand, with some fine-tuning, according to how policymakers weigh the significances between cost and efficiency. Finally, the conceptual methodology in this research can be applied to other areas, such as telecommunications and energy products.

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CHAPTER 1

INTRODUCTION

1.1 Background

"Money" is an important part of our daily life and has been playing a significant economic role in human civilization for a long time. Society accepts the usage of money as the medium of exchange for goods and services, for repaying debt obligations, as well as storage of value in addition to other assets such as gold and land.

At present, "money" can have a variety of forms, such as cash[1], current bank deposits, savings deposits, and time deposits. Moreover, the concept of money may take the form of electronic money, such as phone cards, plastic cards at the food courts, mass transit cards, etc. In any case, the most traditional and ancient form of money is cash, which comprises of coins and banknotes that are most accepted by everyone in society. In general, the currency structure with the banknotes and coins consists of various denominations to accommodate the payment system and to be capable of paying a wide range of amounts with a limited number of monetary items, which provides convenience to payers and payees. Each denomination structure can be identified by two main components. The first component is the structure boundary, which is composed of the lowest coin value, the highest banknote value, and the transition between coins and banknotes. The second component is the series inside the boundary, which implies a number of coins, banknotes, and total number of denominations.

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[1] "Cash" in this study denotes banknotes and coins, and does not include any form of demand deposits or current deposit.

Table 1.1 Coins and Banknotes Denomination Structures in the Main Industrialized Countries and Thailand

Countries	Coins		Banknotes		Total
	Denomination	Number	Denomination	Number	
Canada	0.01-0.05-0.1-0.25-0.5-1-2	7	5-10-20-50-100	5	12
US	0.01-0.05-0.1-0.25-0.5-1	6	1-2-5-10-20-50-100	7	12
UK	0.01-0.02-0.05-0.1-0.5-1-2	7	5-10-20-50	4	11
Euro Zone	0.01-0.02-0.05-0.1-0.2-0.5-1-2	8	5-10-20-50-100-200-500	7	15
Sweden	0.5-1-2-5-10	5	20-50-100-500-1,000	5	10
Norway	0.5-1-5-10-20	5	50-100-200-500-1,000	5	10
Denmark	0.25-0.5-1-2-5-10-20	7	50-100-200-500-1,000	5	12
Switzerland	0.05-0.1-0.2-0.5-1-2-5	7	10-20-50-100-200-500-1,000	7	14
Japan	1-5-10-50-100-500	6	1,000-2,000-5,000-10,000	4	10
Australia	0.05-0.1-0.2-0.5-1-2	6	5-10-20-50-100	5	11
South Korea	1-5-10-50-100-500	6	1,000-5,000-10,000	3	9
New Zealand	0.1-0.2-0.5-1-2	5	5-10-20-50-100	5	10
Thailand	0.25-0.5-1-2-5-10	6	20-50-100-500-1,000	5	11

Source: Adapted from Desjardins Group, 2008: 6.

Table 1.2 Currency Denomination Re-structuring in Thailand since 1981

Year	Coins		Banknotes		Total
	Denomination	Number	Denomination	Number	
1981-1985	0.25-0.50-1-5	4	10-20-100-500	4	8
1986-1992	0.25-0.50-1-5	4	10-20-50-100-500	5	9
1993-1998	0.25-0.50-1-5	4	10-20-50-100-500-1000	6	10
1999-2004	0.25-0.50-1-5-10	5	20-50-100-500-1000	5	10
2005-2011	0.25-0.5-1-2-5-10	6	20-50-100-500-1000	5	11

Considering Table 1.1, it can be seen that the currency denomination structures in various countries may be different. A similar characteristic is that the denomination ranges are based on binary-decimal triplets (1-2-5-10-20-50-…) and fractional-decimal triplets (1-2.5-5-10-25-50-…), which are in line with the decimal currency system[2]. They also include power of ten values such as 1, 10, 100 and 1000, which are easy to use, count, sort and calculate. Moreover, the inside denominations between adjacent power of ten values are factors of the highest value; for example, 10 is divisible by 2, 2.5 and 5. This is another reason why these two structures are applied in the real world, especially the binary-decimal triplets. However, the structure boundary and the series inside the boundary are different across countries, for example, the transition between coins and banknotes, and the number of denominations. Even in each country, the denomination structure is adapted over time. For example, the Thai currency denomination was restructured in the past, as summarized in Table 1.2. The possible factors affecting an appropriate denomination structure for any country at a specific time might be social preferences and behaviors, including economic factors such as price level, electronic payment usage, and shocks (Barry, 1994: 350; Desjardins Group, 2008: 1-2; Kippers, Van Nierop, Paap and Franses, 2003: 485). One example of a shock is the case of hyperinflation in Zimbabwe which, according to the latest change, made the Z$10 billion re-denominated as Z$1. The 10 zeros were not only cut in order to try to slow inflation, but also to make the currency more manageable for the public.

In addition, there will be some disadvantages if the denomination structure is not appropriate for the country. Table 1.3 shows the case of inappropriate structure (denoted as non-optimal later in this study) separated by components.

According to Table 1.3, the disadvantage of the non-optimal structure is that it can be exposed as several types of economic losses, composed of direct and indirect losses. Direct losses are, for example, production, issuing, distribution, and storage costs, whereas indirect losses might come from many patterns, such as inflation costs, user costs, menu costs, and cash payment efficiency loss.

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[2] A decimal currency is a currency where the ratio between the main unit and sub-unit is an integral power of ten for example, 1 dollar = 100 cent, 1 baht = 100 satang.

Table 1.3 The Disadvantages of Inappropriate Denomination Structure by Component

	Too High Value	Too Low Value
1. Lowest denomination	• inefficient payment according to the principle of least effort • impact (possible) on inflation from price rounding up	• difficult to carry and calculate • hoarding phenomenon[3] • tend to generate negative seigniorage (loss)
2. Highest denomination	• psychological impact on inflation • risk of illegal activities and counterfeiting[4]	• inefficient payment according to the principle of least effort • inconvenient in case of large payments
3. Transition between coins and banknotes	• high production costs from producing high-denomination coins (more costly but low demand)	• high production costs from producing low-denomination banknotes (less durability with high demand)
4. No. of denominations	• difficult to calculate, sort, and differentiate • high fixed costs, e.g. menu cost, issuing cost	• non-efficient payment according to the principle of least effort • high production cost from large number of monetary items

In fact, the public seems to accept unavoidably any issued currency denomination structure in their country. However the appropriate structure would facilitate the consumers' cash payments and reduce the costs incurred to producers and issuing authorities[5]. Therefore each country should periodically revise its currency denomination structure so that it is compatible with changes in society and in economic situations.

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[3] For example, 25 and 50-satang coins in Thailand, which lead to artificial demands and non-necessary supply.

[4] For example, Canada removed the $1000 banknote from the circulation.

[5] The authority for banknotes in Thailand is the Note Printing Works, Bank of Thailand, while coins come under the authority of the Royal Thai Mint, Treasury Department, Ministry of Finance.

1.1.1 Cashless Society

Recently, the initiation of a cashless society in the coming decade has been widely discussed among various groups. A cashless society is a culture where cash is used by no one, and all purchases are made by other means of payment, such as credit cards, debit cards, the Internet, mobile phones and smart purses, which are known as electronic payment (e-payment). This idea stems from the fact that e-payment has rapidly grown around the world, including Thailand. Moreover, e-payments are expected to be more efficient and less costly than cash. E-payments therefore are being pushed by responsible authorities to be the major means of payment instead of cash. However, e-payment cannot completely replace cash in terms of social preference and particular characteristics, such as accessibility, traceability, and the simple use of cash without any instruments.

Amromin and Chakravorti (2009: 315) raised the issue of the difference between cash and its alternatives—that is, cash can be used several times without third-party intervention. It is therefore attractive for facilitating illicit transactions, which are difficult to be traced to payers. Moreover, cash can be suited to be a store of value besides the medium of payment. These arguments support why cash is still widely used.

Considering the cash in circulation in Thailand during last six years, as shown in Figure 1.1, it can be seen that the amount of cash in circulation is still rising for both coins and banknotes, and at a positive growth rate. Moreover, Figure 1.2 shows the ratio of cash in circulation to GDP for Thailand, which has been between 10 and 12 percent in last ten years. This implies that cash grows and moves along the GDP and is expected to continue to exist. Consequently, in addition to the promotion of policy towards e-payment, cash management should be considered by determining an appropriate denomination structure compatible with public demand, especially when there are various changes in circumstances such as e-payment usage, price level, and social preferences.

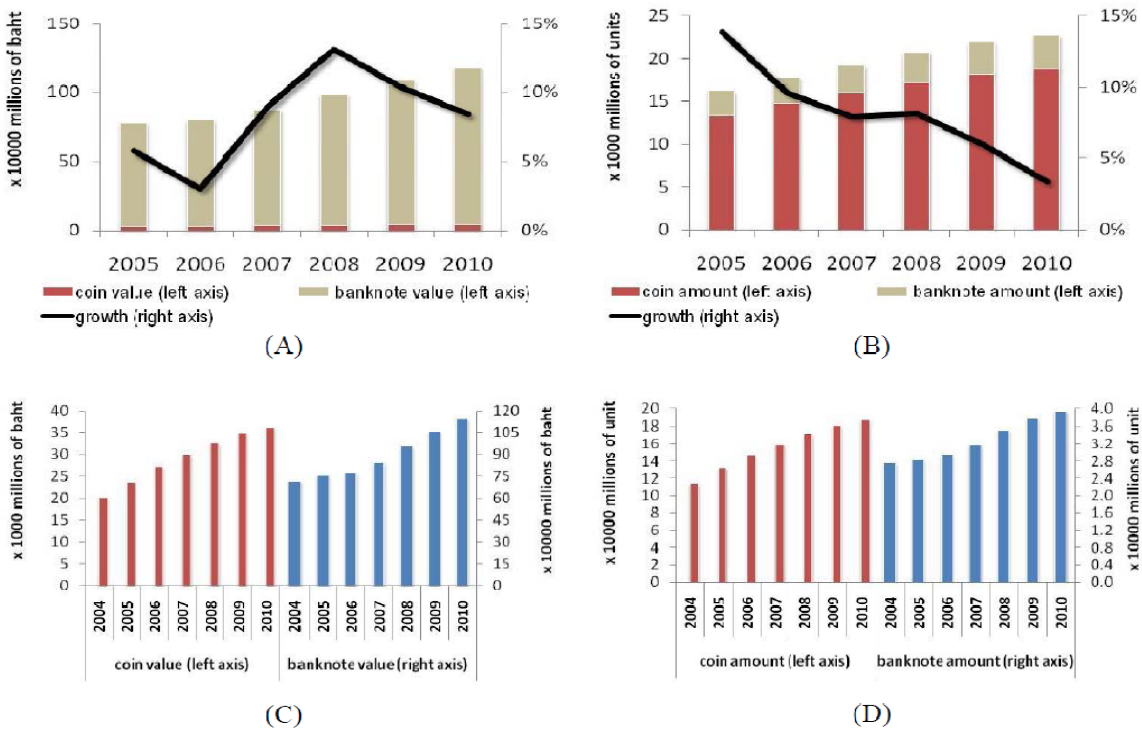

Figure 1.1 Banknotes and Coins in Circulation

Source: Bank of Thailand (BOT): 2011

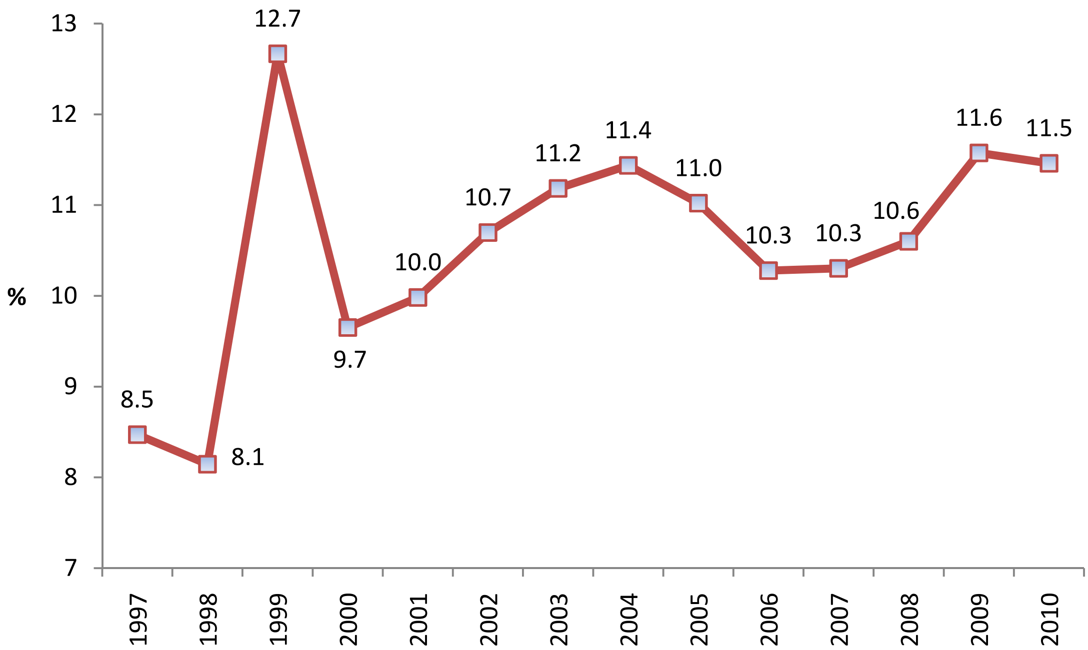

Figure 1.2 Ratio of Cash in Circulation to GDP for Thailand

1.2 Research Question, Objectives, and Contribution

1.2.1 Research Question

According to the changes in the economic situation, such as the rapid growth in electronic payment and continuously higher prices and income level, there might be some potential economic losses from an inappropriate currency denomination structure which lead to the research question: What should the currency denomination structure in Thailand be according to the cost and efficiency perspective? The optimal denomination structure addressed here includes the issue of the lowest and highest denomination, denomination spacing, and the transition between coins and banknotes.

1.2.2 Research Objectives

The intention of this study is to determine the optimal currency denomination structure of Thailand by considering the efficiency of the payment system and the costs incurred by all economic agents. The solution to the optimization problem has been used to verify the current currency denomination structure in Thailand as to whether it is optimal (static analysis). Moreover, it has been further applied to planning for a future currency denomination structure (dynamic analysis).

1.2.3 Contribution of the Research

An optimal currency denomination structure would optimize the retail payment system by increasing the efficiency with which the public can settle cash payments, and help the country to reduce costs for the society. This research could also provide an alternative direction for policy makers to determine the currency denomination structure from an economic optimization perspective. Moreover, this paper will create a specific model and method for verifying the currency denomination structure in Thailand when considering more relevant factors, e.g. e-payment, consumer preferences, payment profile, etc.

1.3 Scope of the Research

To determine the optimal currency denomination structure is a huge task, dealing with both coins and banknotes. This task also includes many components, for example, the lowest value of coins, the highest value of banknotes, the transition between coins and banknotes, and the series inside the boundary. Actually, these four components should be separately determined due to their specific problems and features. For example, of the four components, first, the lowest value of coins may require particularly concern regarding user costs from overly-complicated and wasteful transactions, the loss in the value of coins from the hoarding phenomenon, and the risk to inflation from prices rounding up. Second, the highest value of the banknote has to be determined together with price level, electronic payments, and risk of illegal activities and counterfeiting problems. Third, the transition between coins and banknotes has to be focused on the trade-off between the cost and durability of coins and banknotes together with the price level. Finally, the series inside the boundary, which should be the last decision after the boundary from first three components is known, requires the study of public preferences, the costs incurred from the supply side, and cash payment efficiency.

In this research, the last component, which is the series inside the structure, has been selected as a focal point. Moreover, the study would focuses on only banknote denomination structure.

Why Banknotes? In Thailand, the authorities responsible for coins and banknotes are different. Coins are served and managed by the Royal Thai Mint, Treasury Department, Ministry of Finance, while banknotes are managed by the Note Printing Works, Bank of Thailand. There are also differences in their properties. In general, coins are costly but have high durability; on the other hand, banknotes are cheaper but have a shorter lifespan or less durability. Moreover, large denominations are expected to be used less frequently than lower denominations. As a result, coins are proper for small denominations, which are mainly used for transactions or for medium of exchange purposes with a high velocity of money. Banknotes, on the other hand, are proper for large denominations, which are not only used for transactions but also for value-storing purposes with a lower velocity of money. It can be generally seen that the value of banknotes in circulation is much greater than that of coins. Banknotes are therefore selected to be examined in this research because they are the main component of cash. Examples of the literature focusing on banknotes are the papers of Kohli (1988: 389-399) and Massoud (2005: 3099-3119).

CHAPTER 2

LITERATURE REVIEW

The literature review is divided into two parts. The first part focuses on the optimal denomination range of a currency system in the theoretical frameworks. The second part concerns the literature on the optimal denomination structure of a currency system in practical frameworks for realistic cases by investigating the empirical results from various sources.

2.1 Optimal Range of Currency Denominations in Theoretical Frameworks

It is still unclear how to figure out what denomination is the optimal one. Even though most of the countries use a currency denomination based on the 1-2-5 or 1-2.5-5 system, it cannot be guaranteed that those are optimal. In this part of the study, the literature on the optimal range or spacing between each denomination is focused on. Basically, two concepts of optimal spacing are discussed. First, the currency spacing is optimized based on "the principle of least effort," which can be achieved with the smallest average number of monetary items[6] used in cash payments. Second, determining the optimal spacing would begin from the idea that the number of denominations should be minimized by a method similar to " Bachet's problem[7]."

The first concept was originated by Hentsch (1973: 279-293, 1975: 309-315), who showed that the higher the density of the range, the fewer tokens are needed to make a payment. However, Hentsch compared denomination systems with limited intervals; thus the solution of optimal spacing cannot be clearly concluded. Caianiello, Scarpetta, and Simoncelli (1982: 81-92) then re-examined this issue in the scope of exact payment. They found that the optimal currency range follows the "principle of least effort," which results in the smallest average numbers of tokens in cash

[6] "Monetary items" refer to coins and banknotes. It, hereafter, is denoted as "tokens."

[7] See Appendix A

payments. This can be achieved by setting a denomination space in the same factor or by using a common ratio as a geometric sequence, which is called the "Modular Currency System." Moreover, they confirmed Hentsch's finding, that the average number of units exchanged is an increasing function of the spacing factor. Therefore, the smaller the spacing factor, the smaller the average numbers of tokens in cash payments, resulting in more efficiency. In order to avoid the "one" spacing denomination, a spacing factor of 2 (i.e. 1-2-4-8-…) is considered to be the most efficient. A decade later, Sumner (1993: 13-29) studied the concept of the "principle of least effort" and extended the scope to overpayment using a mathematical model. The results showed that a uniform distribution of denomination spacing with a factor of 3 (i.e. 1-3-9-27-…) was the most efficient and provided a minimum expected number of tokens exchanged in a transaction. We can see that Caianiello et al. and Sumner showed contradictory results under different assumptions. In the case of exact payment, an optimal spacing factor is 2, while the spacing factor of 3 is the best for overpayment and change.

Then, Hove and Heyndels (1996: 547-552) tried to find the optimal spacing factor using Cramer's approach[8] (1983: 299-300). Cramer assumed that the public's payment behavior is efficient, i.e. each payment involves a minimum number of tokens. He used a computer algorithm to determine the average frequencies of denomination. Hove and Heyndels compared the average frequencies of denomination between spacing factors 2 and 3 according to Cramer's approach and found that a currency range with a spacing factor of 2 outperforms the currency range with a spacing factor of 3 according to the "principle of least effort." Therefore, under Cramer's approach, the optimal spacing is indeed two regardless whether overpayment or the return of change is considered.

The second concept of the optimal denomination of currency was originated by Telser (1995: 425-427). He compared the problem of the optimal denomination of currency to Bachet's problem, which is the problem of finding the optimal set of standard weights by seeking the smallest number of weights that will weigh any integer quantity within a given interval on a two-pan balance. In this problem, the

[8] See Appendix B

number of denominations is minimized rather than the number of tokens being exchanged. It was found that the solution is a range with a denomination spacing factor of 2 in the case of exact payment (standard weights in one pan) and 3 in the case of overpayment (standard weights in two pans), which is the more realistic case. Wynne (1997: 221-225) then tested Telser's results using cross-country data on 156 countries. The results showed that the arithmetic mean of the average spacing factor across countries was exactly three, as predicted by Telser. Nevertheless, Wynne's data showed some bi-modality evidence; within the OECD countries, the mode is around 2.2. At the same time, Tschoegl (1997: 546-554) also investigated Telser's results but using the data on 50 countries. The results showed that for many countries the average is "close to three." Across the fifty countries, the average of the averages was 2.60 and 2.62 for coins and banknotes, respectively.

However, Hove (2001: 1015) argued that finding the optimal range of denominations was not the same as solving Bachet's problem because of one critical assumption. Due to the fact that in seeking the optimal set of standard weight, only one weight of each size can be used, while in cash transactions, a payer can use multiple units of one denomination. He concluded that finding the optimal range of denominations should be considered as a multicriteria optimization problem and that "the principle of least effort" should still be given greater weight as a criterion.

The above literature mainly focuses on a theoretical decision regardless of other criteria, such as users' behavior and some implicit costs. As pointed out by Caianiello et al. (1982: 84), besides the minimum condition on the average number of units exchanged, at least two other elements should be taken into consideration. First, for most countries, the denomination structure has to be compatible with the decimal system of currency for easy mental calculation. Second, the spacing factor should not be too low. If this is not the case, practical inconveniences will arise for both the public and currency authorities. Hove and Heyndels (1996: 548) explained the conflict from lowering the spacing factor—it reduces the average number of units needed in a transaction, resulting in a small total number of notes and coins in circulation. It seems that the production and handling costs will be low but adding a new denomination leads to fixed costs of issuance. Therefore, the currency authority faces the trade-off between variable production cost and fixed cost. They also proposed binary-decimal triplets (1-2-5, 10-20-50) or even fractional-decimal triplets (1-2.5-5, 10-25-50) as an option for answering this trade-off problem. Hove (2001: 1020) concluded that the currency systems based on binary-decimal triplets and fractional-decimal triplets appear to be an adequate compromise for three reasons. Firstly, it can be noted that the average multiple of this system equals 2.2, so that its theoretical efficiency remains very close to that of a powers-of-two system. Secondly, this series is compatible with the decimal system. Finally, the total number of denominations is lower than that in a pure powers-of-two system.

For the case of Thailand[9], it can be seen that the denominations in the past included some series of power of two (1-2-4-…), which are optimal according to the principle of least effort. However the system has changed to cover binary-decimal triplets (1-2-5, 10-20-50) or fractional-decimal triplets (1-2.5-5, 10-25-50) to support the decimal currency system. The currency denominations have been developed several times, for example, by removing the 1-satang coin, introducing 2-baht coins, introducing 1000-baht banknotes, and replacing the 10-baht banknote with 10-baht coins.

2.2 Optimal Range of Currency Denominations in a Practical Framework

It can be seen from the current circumstance that most countries around the world have usually applied the currency denomination based on the 1-2-5 series[10]. However, there still are differences in the details of the whole structure concerning the following: the lowest and highest denominations, the number of denominations, the transition between coins and banknotes, and the spacing between denominations.

There is, so far, no exact practical method on how to determine overall optimal denomination. One method is the D-Metric model[11], which was developed by Payne and Morgan (1981: 47). It has been applied to be a re-denomination guideline

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[9] See Appendix C

[10] Two may be replaced by 2.5 in some parts of the range for some countries such as Canada and Thailand.

[11] See Appendix D

for many countries due to its simplicity of use. The model advises when the denomination structure should be modified in terms of the lowest-highest denominations and the transition between coins and banknotes. The model takes into account purchasing power, which is reflected by the average daily pay, for being a main factor to determine the appropriate denomination structure. However, the limitation of this model is the absence of other factors which should be taken into consideration, such as the costs associated with economic agents and users' preferences, including payment habit (Kippers et al., 2003: 13; Mushin, 1998: 255). In addition, Mushin argues that in order to determine the transition between coins and banknotes, the cost and durability of monetary items are the key factors but they are not included in the D-Metric model.

In addition to the D-Metric approach, there might be other methods to determine the appropriate currency denomination structure. In this section, the literature on the practical frameworks are discussed, separated by components, as shown in 2.2.1-2.2.4

2.2.1 The Lowest Denomination

In general, currency denominations with small value are usually used to facilitate cash payments with a high circulation in the system. The main function therefore is to serve as a medium of exchange. According to their characteristics, small denomination currencies have to be coins with higher durability than banknotes. In order to determine an optimal currency system, the lowest currency denomination has to be taken into consideration because too low or too high a value leads to a non-optimal payment system. A lowest denomination that is too low is efficient in terms of the payment system according to the principle of least effort, but there might be some indirect costs incurred to cash users from some wasteful transactions. On the other hand, a lowest denomination that is too high will lead to an inefficient payment system because the number of tokens used for payment is not minimized even though the production cost is expected to be reduced.

The experience of determining the lowest currency denomination in many countries is as follows: Australia stopped making one- and two-cent coins in 1990. New Zealand stopped making them three years before that. France, Norway, and

Britain are among the other countries that have eliminated low-denomination coins. In United States, there is a hot debate on the elimination of the penny or 1-cent coin from the currency system. One reason that supports the idea of retaining the 1-cent coin is the awareness of inflation from higher prices because prices have to be rounded to the nearest 5 cents (Lombra, 2001: 433-434). This is not the case, however, if the rounding method follows a mathematical method with the uniform distribution of prices. By this case, the prices with a final digit ending with 8, 9, 0, 1, or 2 have to be rounded to 0, while the prices with a final digit ending with 3, 4, 5, 6, or 7 have to be rounded to 5. Moreover, the probabilities of prices ending with 0-9 have an equal chance of one-tenth to be rounded up and down. Therefore, getting rid of pennies will not cause the price level to go up; in other words, it will not generate high inflation, which is in line with the explanation by Desjardins Group (2008: 15). Empirically, Lombra (2001: 435) investigated the impact from eliminating the penny from the U.S. coinage system and found that the prices are not uniformly distributed. Thus, elimination of the penny might boost the inflation rate. Nevertheless, it can be seen from his study that the final digits of prices will become more uniformly distributed as the number of items per transaction increases. Chande and Fisher (2003: 515) disagreed with Lombra's results because of the limitation of Lombra's work, which is the absence of a sales tax. Moreover, in order to take advantage of the rounding, a merchant would need to know the different combinations of items purchased. For example, the after-tax prices ending with 8 will be rounded up if the consumer buys one item of good but rounded down if the consumer buys two items. Therefore, the problem of taking advantage of the rounding can be ignored. Chande and Fisher then rejected the hypothesis that rounding will have an effect on prices if the penny is removed from the circulation. Whaples (2007: 140) agreed with Chande and Fisher—that it is unknown how many items consumers purchase from convenience stores. The "rounding tax" is lower as more items are purchased, but information on how many items are purchased was unavailable and many purchases involve more than three items. He also proved that rounding was in fact, on average, done symmetrically. More importantly, the elimination of the penny would not stop prices from being quoted in cents and non-cash payments can still be made in cents.

In the United States, the decision on whether to save the penny is ambiguous. The majority of American people want to keep it. Even though the intrinsic value of the penny is higher than its nominal value (negative seigniorage), the United States Mint still uses pennies and has sought other more economical alloys used for their production. The fact that American society is very conservative, particularly with its symbols, is the one reason that the United States decided not to get rid of pennies from the currency system. On the other hand, Canada has not followed the example of the U.S. Chande and Fisher (2003: 512), using data from the Royal Canadian Mint (RCM), showed that the seigniorage on a penny is negative. In addition, there have also been user costs associated with the inconvenience of the penny. With these two points (negative seigniorage and user cost), and the fact mentioned above that rounding prices to the nearest nickel (5-cent coin) will not be inflationary, Canada should eliminate pennies from its system.

Similarly, Desjardins Group (2007: 1-12) studied whether the penny should be eliminated in Canada. The key of the study is the concern of the estimated cost to financial institutions, retailers, and consumers. The total cost assigned to the corresponding economic agents is 130 million dollars per year. The study concluded that the penny should be retired as quickly as possible. The Desjardins Group pointed out a case study of New Zealand, which was an example of a good practice, and found that the first step for the government to remove the penny from circulation is to convince people that doing so would have no effect on price level or inflation. One year later, the Desjardins Group (2008: 1-25) re-examined all Canadian denominations (coins and banknotes). Their view about the smallest denomination came from their study in 2007, in which they explained more about the purchasing power of a penny, which has continued to decrease, leading to the phenomenon that Canadians increasingly hoard one-cent coins rather than use them to pay for their cash purchases. This hoarding phenomenon will generate an artificial demand for pennies and cause the high production cost. They also analyzed the denomination structure using the D-Metric Model. Concerning the lowest denomination point, the model suggested Canada have removed the penny beginning in 1982 and should remove the 5-cent coin within 20 years. Therefore the government has to stop using 1-cent coins immediately in order to reduce costs for Canadian society and to make the payment system more efficient due to fewer coins being used per transaction.

2.2.2 The Highest Denomination

In general, currencies with a large denomination have a long lifespan because they mainly serve as a store of value and are therefore handled more carefully and less frequently. According to their characteristics, large-denomination currencies should be banknotes rather than coins, with lower production cost and less durability. In order to determine a currency structure, the highest currency denomination has to be considered because it can facilitate cash transactions. If cash payment, on average, is not large in amount, users do not need too high a denomination because it will generate unnecessary costs, such as the cost of producing and issuing. The factors that may be related to the highest denomination are price level and inflation. The higher price level it has, the larger the value the highest denomination should have. However, the rise of alternative means of payments may also lead to less need for a larger value of the highest denomination.

In Jamaica, in order to determine whether a new highest denomination should be introduced, the Bank of Jamaica (2005) proposed the principle where the highest value denomination should not represent more than twenty percent of the number of notes in circulation and not more than sixty percent of the value of notes in circulation. These benchmarks prevent an over-reliance on a single denomination and ensure the cost-effectiveness of a given denomination structure.

The Desjardins Group (2008: 14) proposed three considerations for analyzing whether large denominations should be in a currency system: the usefulness of banknotes for daily cash transactions; the usefulness of banknotes as a store of value; and the usefulness of banknotes as an anonymous financial asset. In fact, Canada used to have a $1000 banknote as the highest denomination but it was removed from circulation. One reason is that it facilitated illegal activities, e.g. drug trafficking, gambling, prostitution, and extortion. At present, the highest currency denomination in Canada is the $100 banknote denomination. The study also applied the D-Metric model to the Canadian system. The model suggested that the Canadian government should add $200 banknotes now, and $500 and $1000 banknotes within 5 and 10 years, respectively. However, with other concerns, such as an increase in e-payments and the possibility of illegal activities, which affects the demand for cash and externality costs, respectively, they recommended waiting another a decade or so before issuing one or more denominations larger than the $100 banknote. In practice, every five years, the government should evaluate the relevance of introducing a $200 banknote, taking into account changes in other payment methods and the perception of merchants as regards counterfeiting risks and the view of crime-fighting authorities.

In the case of Thailand, Balun (2007: 1-26) studied high denomination banknotes larger than 1000 baht. He investigated the impacts of issuing 2000-baht banknotes on stakeholders, divided into four groups: the Note Printing Works, the Banknote Management Division, commercial banks, and cash users (both payers and payees). First, the impact on the Note Printing Works is that the demand for 1000-baht banknotes may be lower. Second, the impact on the Banknote Management Division is a higher cost for either new counting machines or software adjustment. Third, the impact on commercial banks is higher costs and inconvenience in modifying Automatic Teller Machines (ATM) and Cash Deposit Machines (CDM). Last, the impact on cash users is the changes in their behavior, where they may carry more amounts of cash. Furthermore, he also classified the costs and benefits of this policy according to both direct and indirect types. The direct costs are costs of forecasting banknote demands, designing & developing banknotes, supplying new machines, etc. The indirect costs may be incurred from the possibility of illegal activities, the psychological effect of inflation, and the risk of carrying large amounts of cash. The direct benefits are greater convenience in transaction for high-value payments, and lower costs for commercial banks for banknote transportation and management. The indirect benefit is to be a tool for monetary policy. Altogether, quantitative analyses are required to estimate the net benefit of a judgment as to whether the new highest banknote denomination should be introduced into a currency system.

2.2.3 The Transition between Coins and Banknotes

In general, the smaller a banknote's face value, the greater its circulation and the faster it wears out, which explains why currency authorities tend to use durable coins for their small denominations. In fact, the larger denominations have a longer lifespan because they mainly serve as a store of value and are therefore handled more carefully and less frequently. Conversely, small denomination banknotes have a shorter lifespan due to less careful handling and wider circulation. Therefore, determining the transition between coins and banknotes requires knowing when to replace banknotes with coins of the same face value in order to arrive at a compromise between related costs and durability.

Barry (1994: 350-352) explained the case of New Zealand, that replacing 1- and 2-dollar banknotes with 1- and 2-dollar coins had been done for cost-effectiveness reasons. In the case of Jamaica, the Bank of Jamaica have done the same; that is, whenever a banknote ceases to be cost-effective relative to the face value of the denomination, the Bank will consider replacing it with a coin of similar face value. In its analysis of the cost-effectiveness of the note, the Bank takes account of both the cost of production and the average useful life of the banknote relative to the coin. A banknote may also be replaced with a coin of similar value when the Bank is realigning its denomination structure after the introduction of a higher value note, as happened in 1994 when the $5 banknote was replaced by the $5 coin following the introduction of the $500 note. The Desjardins Group (2008: 7) additional explained that durability and production costs are two factors behind the decision about whether to issue coins or banknotes. The study also compared the situation of Canada to the D-Metric model, which assigned the transition between coins and banknotes to be within the range of D/50 and D/20, where D is the amount of the average day's net pay. The model suggested that the Canadian government replace the $5 banknote with a $5 coin within ten years. Similarly for Thailand, the next plan suggested by D-Metric is to replace 20-baht banknotes by coins with the same face value in the next 8-10 years (Balun, 2007: 23-24).

2.2.4 The Spacing between Denominations

We can see that the 1-2-5 currency denomination series is generally accepted as the optimal denominational series of banknotes and coins and is more common than the 1-2.5-5 series (Kippers et al., 2003: 485). These two series are a compromise between the efficient payment concept according to the principle of least effort and the practical use concept of the decimal system. At present, most countries use either one of the two series or a combination of them, e.g. 25-satang and 20 baht for Thailand. The Desjardins Group (2008: 6) pointed out that an industrialized country typically has approximately 12 denominations, broken down as follows: five to seven coins and four to seven banknotes. In the past few years, some countries have reduced their denominations to fewer than 12 due to demonetization or non-use of their low-denomination currency (as occurred in Australia, New Zealand, Sweden, and Norway). Other countries, such as Japan and the United Kingdom, prefer to keep a limited number, that is four banknotes, in circulation. It should be noted that the Eurozone is an exception among industrialized countries, with 15 denominations (eight coins and seven banknotes). They explained further that the number of coins and banknotes for a given economy depends on several factors, such as the extent of household consumption, the level of interest rates, changes in price indices, average personal income tax rate, the number of ATMs, as well as the popularity of electronic payment and methods of use (for example, credit cards, debit cards, and, eventually, smart cards). However, the greater the number of denominations, the greater is the cost of keeping them in circulation for the currency issuing authorities. The existence of too many denominations can make it difficult for users to differentiate between them when making purchases. In other words, having too many denominations will probably reduce their use, which largely explains the phenomena of hoarding by users and the resulting increased production costs. Sometimes, the new currency denomination has to be introduced between two denominations, such as the case of New Zealand when launching the $50 banknote to fill the sizable gap between the banknote series $20 and $100 denomination. Barry (1994: 350) indicated that the reason for this was the double-digit inflation in the 1970s, which created more demand for the $50 banknote, which was the second highest denomination at that time (the highest was $100).

On the other hand, some countries need to eliminate some currency denominations in order to make the structure more compact. In Jamaica, for example, to determine whether a denomination should be withdrawn, the Bank of Jamaica (2005) examined two indicators: the demand for the denomination and the usefulness of the life of the denomination, which refers to the period of time the unit of currency is in active circulation. The denomination should be removed if both indicators are extremely low and short. However the one-cent coin will continue to be an exception of this policy, as its inclusion in the denomination structure is a legal requirement and it is a base unit for the entire currency structure.

Another example of currency denomination modification is Euro money. For example in the Netherlands, the transition from the Guilder to the Euro involved a transition to a different denominational structure. The Guilder banknotes of 1000, 250, 100, 50, 25 and 10 were replaced by Euro notes of 500, 200, 100, 50, 20, 10 and 5, which in fact are rather similar in value (EUR 1 = NLG 2.20371), except for the 200-Euro note. This banknote is new, as the Guilder range did not include any banknote with a comparable value. Furthermore, the new Euro coins consist of the following denominations: 2, 1, 0.50, 0.20, 0.10, 0.05, 0.02 and 0.01, and this range involves two more coins than the Guilder range used to have, which were 5, 2.50, 1, 0.25, 0.10 and 0.05 Guilders. The 20-Eurocent coin is new, as it amounts to about 50 Guilder cents, as well as the 1-Eurocent coin, which is about 2 cents in Guilders. Kippers et al. (2003: 505) examined this policy and found that the transition to Euro notes and coins did not seem to make a difference for the Dutch paying public, although the new 1-2-5 range of the denominations is perhaps a little more efficient than the old 1-2.5-5 range of Guilder notes and coins. However, he used efficient payment as the criterion to obtain a better denomination structure. This model is based on “the principle of least effort.” If individuals would behave according to this principle, each amount would be paid so that the number of notes and coins exchanged would be minimized. He concluded that the payment system would benefit from the removal of the 1-cent and 2-cent coins, which would make the Euro range more efficient. In addition, his computations suggested that the removal of a 10-Euro or 100-Euro banknote would cause the range to be less efficient, but this loss in efficiency is small.

In sum, each country may have undertaken currency re-denominations at various times and for different reasons. The countries, for example, include Afghanistan (2002), Germany (1923, 1948), Argentina (1970; 1983; 1985; 1992), Bolivia (1963, 1987), Brazil (1967, 1967, 1970, 1986, 1989, 1990, 1993, 1994), China (1955), South Korea (1962), Mexico (1993, 1996), Ghana (2007), Israel (1948, 1960, 1980, 1985), Turkey (2005), and Angola (1995, 1999).

The re-denominations could be prepared by considering the lowest and highest denominations, assigning the transition between coins and banknotes, choosing a denomination spacing system, and making decisions on which denominations should be eliminated from and introduced into the series.

CHAPTER 3

CONCEPTUAL FRAMEWORK AND THEORETICAL MODEL

3.1 Overview and Framework Diagram

In the past, there has been no study to find out the completely optimal denomination in a whole structure which contains several sub-components, for instance, the lowest denomination, the highest denomination, and the transition between coins and banknotes. The denomination structure is usually investigated by considering separately each component, e.g. should the 0.25-baht coin be eliminated from the currency system, when should 20-baht banknotes be replaced by 20-baht coins, and is it time to introduce 200-baht /or 2000-baht banknotes in the currency system? The examples and case studies from other countries were discussed earlier in the second section of the literature review.

The purpose of this research is to determine optimal denomination structure by applying total cost minimization with: 1. maximized utility explained by currency demand and, 2. high cash payment efficiency according to the principle of least effort. With this method, both supply and demand can be determined covering all economic agents. The demand side, dealing with payers, payees, and commercial banks, is considered in terms of utility and cash payment efficiency. The supply side, dealing with the Central Bank for banknotes, the Treasury Department for coins, and also commercial banks[12], is mainly considered in terms of costs.

According to the background above, the conceptual framework is mainly based on the cost minimization approach, composed of many components, as shown in Figure 3.1.

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[12] Commercial banks are concerned and located on both supply and demand sides because they are intermediaries between monetary-issuing authorities and the public. They need not only an efficient denomination structure to facilitate their financial transactions but also low operating costs, such as costs incurred from distribution and ATM modification.

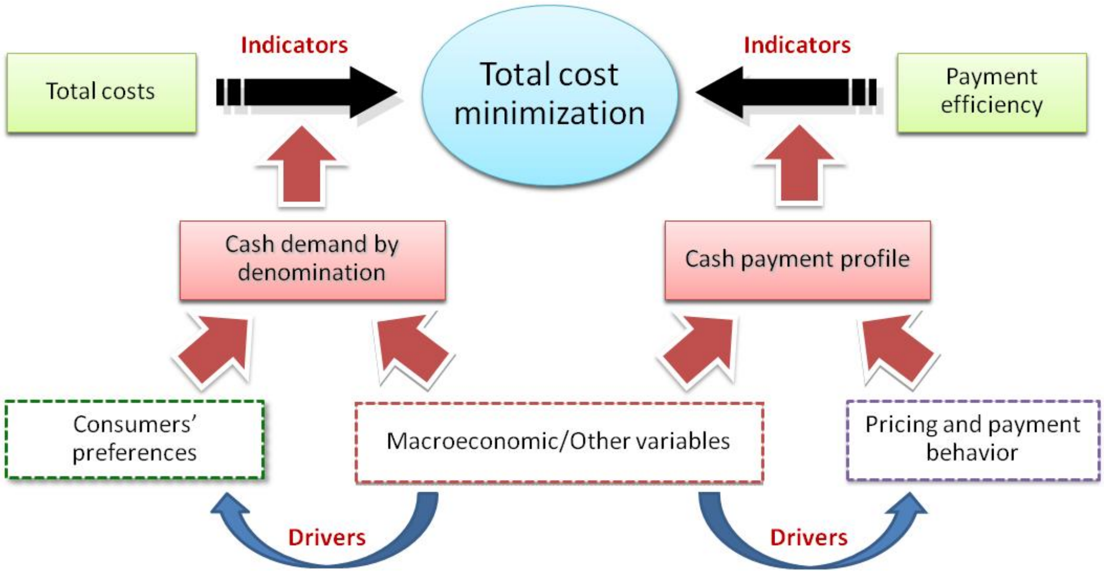


Figure 3.1 Conceptual Framework of the Research

According to Figure 3.1, the objective function to be optimized is total cost with the consideration of payment efficiency. In order to analyze total costs, cash demands by denomination are required to identify cash in circulation and the production plans for currency by denomination, while a cash payment profile is needed for weighted average calculation in order to provide more precisely estimated cash payment efficiency. However, cash payment efficiency is an intangible issue, which people can not directly realize, but some parts may be already embedded in the demand estimation on the basis of the characteristic model. In addition, there are some characteristics of banknote denominations, such as average spread and number of denominations, that partially relate to cash payment efficiency. Therefore, in order to avoid double counting, payment efficiency should be a constraint rather than an objective to be optimized. As a final point, the total cost would be the ultimate value to be minimized under a cash payment efficiency constraint. According to the scope of the research explained in section 1.3, that focus will be placed on the banknote denomination structure, from this point forward, the scope of the denomination structure will be narrowed from "currency" or "cash" denominations (of coins and banknotes) to "banknote" denominations.

In sum, there are four essential components: banknote demand by denomination, total costs, cash payment profile, and cash payment efficiency. Each component may require sub-components and some related information. For example, banknote demand by denomination was originally founded on consumers' preferences, whereas the cash payment profile was constructed from the payment behavior of consumers and pricing by merchants. Moreover, there will be some macroeconomic and other variables which affect banknote demand by denomination and cash payment profile. The details of each component are separately explained in the next sections (3.2-3.5).

3.2 Banknote Demand by Denomination

According to the method used here to determine the optimal banknote denomination based on the minimum cost perspective and the hypothesis that different denomination structures cause different demands for each denomination, the banknote demand by denomination, which reflects the consumers' preferences regarding denomination structures, is then needed so that total cost can be estimated and compared among denomination structure alternatives.

Actually, there are several ways to derive demand function, such as consumer surveys and econometric models using historical data. In the case of denomination structure, which acts as a good in consumption theory, there are various preferences of people depending on age, career, etc. to identify their satisfaction with the denomination structure. Businessmen that are familiar with e-payments, such as credit cards, may prefer a compact structure, with not too many denominations, in order to avoid receiving various coins and banknotes from a currency system. On the other hand, merchants may prefer a full structure, with a lot of denominations, to facilitate their transactions. The survey approach therefore has to be carefully taken considered, with a complicated design in order to be able to respond to social preferences. The respondents must be thoroughly sampled, covering sex, age, career, income, and area, in order to reflect all types of consumers. Moreover, the problem of bias from surveys also raises the level of complexity from the survey approach to obtain information on demand. Consequently, this study avoids the use of surveying to estimate demand function.

In practice, the Banknote Printing Work usually estimates banknote demand share using historical data to analyze a time trend according to certain patterns, such as moving average. The strength of the current method is the ease with which it obtains acceptable results, but there is no absolute determination regarding the factors affecting demand. Consequently, it is difficult to explain demand shares when, 1. the currency denomination structure changes (e.g. there are new denominations issued in circulation or some denominations are eliminated from the structure), 2. some macroeconomic factors change (e.g. GDP, price level and e-payment usage, etc.), and 3. the characteristics of monetary items are modified (e.g. size, color, material used, weight), and especially when all three changes above are happen simultaneously.

The alternative approach is the use of historical data, which is the currency in circulation, to reflect the social preferences regarding currencies by denomination. They are then applied to the econometric model, together with the important factors to specify the demand share for each currency and to estimate currency demand by denomination. It can be seen that with this approach, the aggregate data would be applied to obtain the aggregate demand, which is less complicated than the use of the survey approach. This approach is the concept of the "characteristic model," which extends the traditional demand estimation to the version in which the characteristics of goods are taken into the model.

3.2.1 Characteristic Model

The characteristic model was originally proposed by Lancaster (1966: 132-157). He explained consumer demand using different methods from traditional theory. Lancaster's approach stated that the characteristics of goods are demanded by consumers, not the goods themselves. For example, consumers do not demand apples or durian in themselves but rather the flavors and nutrients in them.

Lancaster stated that the traditional approach cannot explain how demand will be affected by changes in the goods' characteristics or how new products fit into preference patterns over existing goods. The characteristic model allows us to determine the demand for goods with some changes, new products, and some differentiated products that have common characteristics. The characteristic approach has been applied to many areas in economic studies. The various applications of the characteristic approach can be shown as follows:

Kohli (1988: 389-399) used the characteristic model to explain the six Swiss banknote denominations and found that the six characteristics of banknotes can describe the currency mix that the public wishes to hold. The model also was used to predict the demand for a Frs. 200 banknote, a denomination which does not exist at the moment in Switzerland.

Kohli and Morey (1990: 55-67) modeled the United States import demand for foreign crude oil by region of origin using the characteristic approach. Import demand was estimated using data from eight major oil suppliers to the U.S. and five characteristics of oil were taken into account. With this approach, the costs (or the benefits) from the changes in foreign crude oil characteristics can be evaluated. It is also used to predict the import demand from regions not included in the sample.

Marcin (1992: 119-124) proposed a conceptual framework to analyze the demand for composite wood products according to the characteristic model. With this approach, it is possible to examine the demand for alternative wood/non-wood combinations, new composites with various raw materials, or composites with some changes in their properties that can be obtained with alternative blends of materials.

In this study, the characteristic approach is applied to determine banknote demand share by denomination. This approach allows for the prediction of demand share for banknote denominations that have never existed, which is beneficial to a forward-looking study as represented by this research. Together with the total banknote demand figure, the demand for banknote by denomination can be obtained. If equilibrium is assumed, banknote demand is expected to be equal to banknote supply and finally implies banknotes in circulation by denomination.

3.2.2 Banknote Demand with the Characteristic Model

In order to match the selection of a banknote denomination to consumer choice in microeconomics, we will first introduce consumption theory, which explains how consumers make decisions in consuming. The theory relates to the utility maximization problem under limited money income, which is a budget constraint.

Comparing the analysis of the demand for banknotes by denomination, the basis of utility maximization can be applied in order to derive the demand function, where the goods to be consumed are the banknotes needed by the public. The constraint for the problem is the given total banknote demand, which relates to the money supply controlled by a central bank. The main difference is that the utility in this case represents the preferences of the public or social utility. The important hypothesis is that the banknotes in circulation in the past relate to the aggregate demand for banknotes, which automatically comes from social utility maximization. Therefore, the historical data on banknotes in circulation can be used to analyze the demand for banknotes by denomination. Furthermore, it is assumed that given their total demand for currency, economic agents determine their demand by denominations by maximizing utility in terms of convenience, security, and so on, in which the public gains from a particular currency mix.

Table 3.1 The Comparison between the Demand for General Consumption Goods and Banknote Denominations

Consumption goods	Banknotes by denomination
• Individual utility	• Social utility
• Money income	• Currency demand = Currency supply
• Good #i	• Denomination #i
• Price of good #i	• Value of denomination #i
• More is preferred to less	• More is preferred to less (to make sure that they are enough for the public)

Let

F_i = the face value of denomination i; i=1, 2, 3, …,I e.g. $F_1 = 2$, $F_2 = 5$

Q_i = the amount of banknote denomination with a face value F_i

M = total banknote demand which is fixed

S_i = share of banknote demand of denomination with face value F_i

$U(\cdot) = U(Q_1, Q_2, \ldots, Q_I ; h_1, h_2, \ldots, h_I)$

h_i = a parameter associated with Q_i and can be explained by K characteristics

In other words, h_i is a function of the K characteristics:

$h_i(\cdot) = h(a_{1i}, a_{2i}, \ldots, a_{Ki})$

a_{ki} = the k^{th} characteristic of banknote denomination with face value F_i

To derive the demand function, one should begin with the utility function, which is the objective to be maximized. Two functional forms are selected to be the learning cases.

1) CES utility function

$U(\cdot) = \Sigma\, Q_i^{(\sigma-1)/\sigma} h_i$

The optimization problem can be set up as follows:

Max $U(\cdot) = \Sigma\, Q_i^{(\sigma-1)/\sigma} h_i$ s.t. $\Sigma F_i Q_i = M$

The first order condition yields the solution in the share form:

$$S_i = \frac{F_i \cdot Q_i}{\sum_{i=1}^{I}(F_i \cdot Q_i)} = \frac{h_i^{\sigma} \cdot F_i^{1-\sigma}}{\sum_{i=1}^{I}(h_i^{\sigma} \cdot F_i^{1-\sigma})} \quad (3.1)$$

2) Cobb-Douglas utility function

$U(\cdot) = \sum Q_i^{h_i}$

The optimization problem can be set up as follows:

$U(\cdot) = \sum Q_i^{h_i}$ s.t. $\Sigma F_i Q_i = M$

The first order condition yields the solution in the share form:

$$S_i = \frac{F_i \cdot Q_i}{\sum_{i=1}^{I}(F_i \cdot Q_i)} = \frac{h_i}{\sum_{i=1}^{I}(h_i)} \quad (3.2)$$

Even though h_i are normalized by Σh_i to guarantee the constant return to scale of utility function, the solution does not change because the ratio of indices is still the same.

We assume a semi-log functional form (log-lin) to approximate h(•).

Hence, $\ln h(i) = \alpha_0 + \Sigma\, \alpha_k a_{ki}$

We obtain $h(i) = e^{\alpha_0 + \Sigma \alpha_k a_{ki}}$

Note that if all characteristics (a_{ki}) are taken into account, α_k will be equal for all denomination determination (Kohli, 1988: 391; Kohli and Morey, 1990: 56). On the other hand, if some characteristics are omitted, α_{ki} should be replaced because each denomination i will have a different α_k. However, in this research, it is assumed that all characteristics are taken into account and that α_k is constant for all demand share equations (S_i).

According to equation (3.1), which is the case of the CES utility function:

$$S_i = \frac{e^{\sigma\alpha_0 + \sum_{j=1}^{J} \sigma\alpha_j a_{ji}} . {F_i}^{1-\sigma}}{\sum_{i=1}^{I} (e^{\sigma\alpha_0 + \sum_{j=1}^{J} \sigma\alpha_j a_{ji}} . {F_i}^{1-\sigma})}$$

$$= \frac{e^{\sum_{j=1}^{J} \sigma\alpha_j a_{ji}} . {F_i}^{1-\sigma}}{\sum_{i=1}^{I} (e^{\sum_{j=1}^{J} \sigma\alpha_j a_{ji}} . {F_i}^{1-\sigma})}$$

$$= \frac{e^{\sum_{j=1}^{J} \rho_j a_{ji}} . {F_i}^{1-\sigma}}{\sum_{i=1}^{I} (e^{\sum_{j=1}^{J} \rho_j a_{ji}} . {F_i}^{1-\sigma})} \quad \text{where } \rho_j = \sigma\alpha_j \text{ for } j = 0, 1, 2, \ldots, J \qquad (3.3)$$

According to equation (3.2), which is the case of the Cobb-Douglas utility function:

$$S_i = \frac{e^{\alpha_0 + \sum_{j=1}^{J} \alpha_j a_{ji}}}{\sum_{i=1}^{I} (e^{\alpha_0 + \sum_{j=1}^{J} \alpha_j a_{ji}})}$$

$$= \frac{e^{\sum_{j=1}^{J} \alpha_j a_{ji}}}{\sum_{i=1}^{I} (e^{\sum_{j=1}^{J} \alpha_j a_{ji}})} \qquad (3.4)$$

The share equation (3.3) is more complicated than equation (3.4) but it allows us to identify σ, which implies the value of a constant elasticity of substitution (according to a property of the CES function).

3.2.3 Application

In order to apply the characteristic model to this research, the banknote denominations are identified to be goods in the model. The CES functional form is selected, which is in line with equation (3.3) as the model to be examined. This is motivated by the fact that it is well known and easily allows for the incorporation of characteristics (Kohli and Morey, 1990: 56). However, in this study, the model has been a little bit adjusted; that is, the characteristics are divided into two main categories: individual and common characteristics. The individual characteristics refer to the features that vary for each denomination, such as size, color, and average spread. The common characteristics refer to the features in which all denominations face, which can be subdivided into two sub-categories: i) macroeconomic variables, such as GDP, and ii) overall structure, such as the number of banknote denominations. To take the common factors into the model, interaction terms are needed to distinguish the effect of those factors on demand share by denomination. Without interaction terms, the common factors will be useless because they will equally affect all share equations (the common characteristics in numerator and denominator are cancelled out). The interaction terms in this study would be the multiples of the face values.

Let

S_i = demand (in value) share for banknote denomination i ; i = 1,2,3, ..., I

A_{ji} = individual characteristic j for banknote denomination i; j = 1,2,3, ..., J

B_k = common characteristic k for all banknote denominations; k = 1,2,3, ..., K

F_i = face value of existing banknote denomination i

α_j = coefficient of individual characteristic j

β_k = coefficient of common characteristic k

σ = elasticity of substitution

Referring to (3.3), demand share for banknote denomination n (S_n) is

$$S_n = \frac{(e^{\sum_j \alpha_j A_{jn} + \sum_k \beta_k B_k F_n}) \cdot F_n^{1-\sigma}}{\sum_{i=1}^{I} [(e^{\sum_j \alpha_j A_{ji} + \sum_k \beta_k B_k F_i}) \cdot F_i^{1-\sigma}]} \qquad (3.5)$$

3.2.4 Comparative Statics

In this section, comparative statics are applied to determine the impacts of relevant factors on banknote demand share. The impacts are categorized into three groups, which are own impact, cross impact and impact of common factors.

3.2.4.1 Own impact which is the impact of individual characteristic j of denomination n on demand share for banknote denomination n):

$$\frac{\partial S_n}{\partial A_{jn}} = \frac{\alpha_j \cdot \left[\left(e^{\sum_j \alpha_j A_{jn} + \sum_k \beta_k B_k F_n}\right) \cdot F_n^{1-\sigma}\right] \cdot \left[\sum_{i=1;\, i \neq n}^{I} \left[\left(e^{\sum_j \alpha_j A_{ji} + \sum_k \beta_k B_k F_i}\right) \cdot F_i^{1-\sigma}\right]\right]}{\left[\sum_{i=1}^{I} \left[\left(e^{\sum_j \alpha_j A_{ji} + \sum_k \beta_k B_k F_i}\right) \cdot F_i^{1-\sigma}\right]\right]^2}$$

3.2.4.2 Cross impact which is the impact of individual characteristic j of denomination m on demand share for banknote denomination n):

$$\frac{\partial S_n}{\partial A_{jm}} = \frac{(-\alpha_j) \cdot \left[\left(e^{\sum_j \alpha_j A_{jn} + \sum_k \beta_k B_k F_n}\right) \cdot F_n^{1-\sigma}\right] \cdot \left[\left(e^{\sum_j \alpha_j A_{jm} + \sum_k \beta_k B_k F_m}\right) \cdot F_m^{1-\sigma}\right]}{\left[\sum_{i=1}^{I} \left[\left(e^{\sum_j \alpha_j A_{ji} + \sum_k \beta_k B_k F_i}\right) \cdot F_i^{1-\sigma}\right]\right]^2}$$

3.2.4.3 Impact of common factors which is the impact of common characteristic k on demand share for banknote denomination n):

$$\frac{\partial S_n}{\partial B_k} = \frac{\beta_k \cdot \left[\left(e^{\sum_j \alpha_j A_{jn} + \sum_k \beta_k B_k F_n}\right) \cdot F_n^{1-\sigma}\right] \cdot \left[\sum_{i=1}^{I} \left(e^{\sum_j \alpha_j A_{ji} + \sum_k \beta_k B_k F_i}\right) \cdot F_i^{1-\sigma} \cdot (F_n - F_i)\right]}{\left[\sum_{i=1}^{I} \left[\left(e^{\sum_j \alpha_j A_{ji} + \sum_k \beta_k B_k F_i}\right) \cdot F_i^{1-\sigma}\right]\right]^2}$$

What can be learned from the comparative statics is the following:

1) Considering the individual characteristic j, the marginal effect of denomination m's characteristic on demand share for banknote denomination n equals the marginal effect of denomination n's characteristics on demand share for banknote denomination m, which can be stated as follows:

$$\frac{\partial S_n}{\partial A_{jm}} = \frac{\partial S_m}{\partial A_{jn}} \tag{3.6}$$

2) When looking at characteristic j across denominations, it can be considered in two dimensions:

(1) $\sum_i \frac{\partial S_n}{\partial A_{ji}} = 0$ (3.7)

which shows that the impacts on demand share for denomination n resulting from the characteristic j of each denomination i add up to zero.

(2) $\sum_i \frac{\partial S_i}{\partial A_{jn}} = 0$ (3.8)

which shows that the impacts from the characteristic j of denomination n on demand share for each denomination i add up to zero.

In the case of 3-denomination structure (with denomination #1, #2 and #3) and considering denomination #1 and banknote size to be a characteristic of concern, the sample interpretations are given as follows:

According to (3.7), it can be explained that the impacts on demand share of banknote denomination #1 from the size of banknote denomination #1 itself, #2, and #3 are summed to be 0. In another illustration, if banknote sizes increase equally in all denominations, there is no impact on demand share for all banknote denominations because the net effect will be equal to zero.

According to (3.8), it can be explained that the impacts of the size of banknote denomination #1 on demand share of banknote denomination #1 itself, #2, and #3 are summed to be 0. In another illustration, if the size of banknote denomination #1 increases, the changes in demand shares of denomination #1, #2, and #3 will offset each other. This is a simple logic in addition to a mathematic explanation if we realize that the sum of demand shares is 1 or 100%.

It can be seen that zero-sum conditions exist for both dimensions.

3) When looking at common characteristic k, zero-sum marginal effects on banknote demand shares for all denominations do exist. In other words, $\sum_i \frac{\partial S_i}{\partial B_k} = 0$, which show a similar implication as (3.8). It can be explained that the impacts from the common characteristic k, such as GDP per capita on demand share for each denomination i, are summed to be zero. Again, this is a simple logic due to the sum of demand shares being controlled to be 1 or 100%.

4) The coefficients can be interpreted as follows according to comparative static derivation:

(1) Own impact of individual characteristics:

The impact of characteristic j of denomination n on its demand share depends on α_j. The positive sign of α_j shows a positive relationship between change in characteristic and change in demand share, and vice versa.

(2) Cross impact of individual characteristics:

The impact of characteristic j of denomination m on demand share of another denomination n depends on α_j. The positive sign of α_j shows a negative relationship between change in characteristic and change in demand share, and vice versa.

(3) Impact of common characteristics:

The analysis of the impact of common characteristics is more complicated to interpret because we have to determine the interaction terms in the model. The impact of characteristic k on demand share of denomination n depends on β_k and its interaction term in the equation of demand share of denomination n, which is its face value (F_n). The positive sign of β_k tends to favor high-denomination banknote, while the negative sign of β_k tends to favor the low-denomination banknote.

(4) Elasticity of substitution:

The elasticity of substitution among denominations can be interpreted from σ, which is assumed to be constant according to the CES utility function.

Note that Leontief, linear, and Cobb-Douglas functions are special cases of the CES function. That is, in the limit as σ approaches 1, we get the Cobb-Douglas function; as σ approaches positive infinity we get the linear (perfect substitutes) function; and for σ approaching 0, we get the Leontief (perfect complements) function.

3.3 Total Cost Structure

According to Cost-Benefit Analysis (CBA), direct and indirect costs have to be considered in total cost, while direct and indirect benefits are also taken into account. In this research on denomination structure selection, the benefits of structure are ignored because it is assumed that all scenarios of denomination structures yield equally benefits in the role of money, i.e. to facilitate and optimize the payment system and serve three main functions of money: as a medium of exchange, as a unit of account, and as a store of value. For the other indirect benefits of each denomination structure, they are determined to be a reflection of indirect cost. For example, of the full denomination structure with many denominations inside, the cost from the difficulty to memorize, calculate, and carry is equivalent to the benefit from the exact payment and minimum tokens used for transaction. Therefore, the benefit was not focused on, and only costs were compared among all scenarios of the denomination structure.

As discussed earlier, that the objective value to be optimized is cost dealing with banknotes, the main types are the costs incurred by the Bank of Thailand and commercial banks, such as production costs, issuing costs, storing costs, and distribution costs.

Considering the total cost structure, as shown in Figure 3.2, the diagram illustrates only the total costs that depend on the banknote denomination structure. They exclude the costs that are not affected when the structure changes, which are fixed regardless what the structure is, for example, the administration cost of the production unit and the salary of employees that work at note printing works. The reason why the latter costs are ignored is because total costs are applied for comparison purposes in order to find the structure which yields the minimum cost. The elimination of equally-fixed costs will make the cost in each scenario be deducted in the same amount, which does not affect the optimal solution. Therefore the realization of this cost type is not necessary. This would be convenient for empirical analysis in the sense that some secure and confidential data can be overlooked.

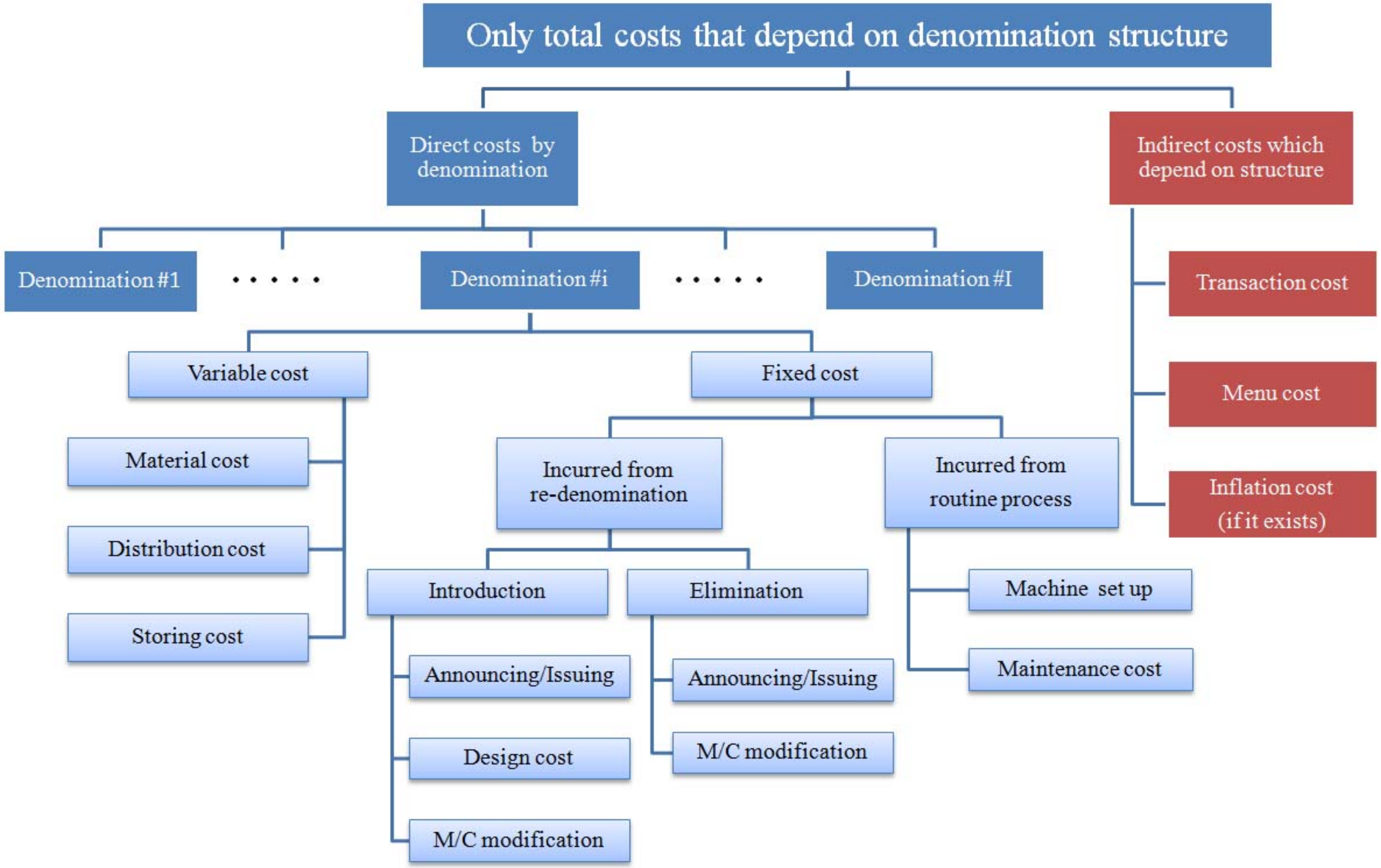


Figure 3.2 Total Cost Structure

Total costs can be categorized as direct and indirect costs. Direct costs are the expenditures directly incurred from the activities dealing with money transfer before the cash is in the hands of users, for example, production, issuing, distribution, and storing costs. They comprise the direct costs of all denominations. The direct costs of each denomination are subdivided into variables and fixed costs. The main economic agents that absorb these direct costs are producers and distributors, which are the Central Bank, the Treasury Department, and commercial banks.

The variable costs mainly come from production and distribution, which depend on the amount of banknotes produced, for example, material costs, transportation costs, and storing costs, whereas fixed costs are subdivided into two types: first, general fixed costs from routine processes such as production, storing and maintenance; second, fixed costs from re-denomination (introduction and elimination or both), such as the cost of announcing, banknote issuing, and machine modification, for example, the printing press and the ATM. However, the costs from producing new-denomination banknotes and destroying old-denomination banknotes will be included in the variable costs for normal processes (old and unused-denomination banknotes do not need to be taken out of circulation; they can still be used afterwards).

Besides the direct costs mainly incurred by central and commercial banks, there are indirect costs, as mentioned earlier. Indirect costs are the implicit costs incurred from a denomination structure, especially when it is modified or re-structured. They may be indirectly incurred from agents' inconveniences and some negative impacts according to changes in the structure. Examples of indirect costs are transaction costs from too few denominations in a structure and inflation costs from too high highest banknote denominations (if any exist). There may be other user costs regarding memorizing, calculating, and carrying, which are also categorized as indirect costs.

In summary, in order to determine the total cost of banknotes applied to this research, five cost components are of concern, which are:

1) Total variable cost
2) Total general (routine process) fixed cost
3) Fixed costs from the introduction for new denominations

4) Fixed costs from the elimination of existing denomination

5) Indirect costs which are implicitly incurred by all economic agents, including cash users (payers and payees)

However, in the model specification, the cost function takes into account only the first four categories, which do not include indirect costs. Therefore, the cost estimated in this study does not provide the exact total social cost, but it can be applied for comparison purposes in order to find out the minimum cost scenario that represents the optimal solution.

Why Remove Indirect Costs from the Cost Function? It can be seen that people cannot directly select the denomination structure themselves and they have to accept the denomination structure and re-structure decided by the Central Bank (for banknotes) and Treasury Department (for coins). However, the users' preferences can be revealed by the characteristic approach, which takes into account structure and currency characteristics. Therefore, it already reflects the user costs mentioned above in the estimated demand function. In other words, some user costs are partially embedded in the demand function derived by a characteristic model. This is one reason why user costs are not required in a cost function to avoid double counting.

Regarding inflation costs, there are many arguments as to whether denomination structure causes inflation. Three structure components have been discussed as a cause of inflation. The first component is the lowest coin denomination. The idea that getting rid of the lowest denomination will cause inflation has been debated. One side believes that if it does, prices will be rounded up, leading to inflation. On the other hand, the other side argues that there will be no inflation problem if the prices are mathematically rounded up and down with equal chances (symmetry). The second component is the highest banknote denomination. The point that launching the highest denomination will cause inflation is not clear. There might be some psychological motivation for the use of cash for payments, especially when there are high denomination banknotes in one's wallet. Currencies are then required to be more in circulation, leading to the higher money supply and inflation later on. However, the research by Franses (2006: 752) concludes that inflation causes the highest denomination to be increase but an increase of the highest denomination does not cause inflation. The last component is the transition between coins and banknotes. It has been explained that replacing the lowest banknote denomination by a new highest coin denomination with the same face value will cause inflation due for psychological reasons. For example, some people feel that banknotes are more valuable than coins of the same denomination; therefore, they use coins more for tips, shopping, and making donations. These two examples are cases which may lead to inflation.

However, the focus only on banknote structure and the assignment of currency denomination boundary[13] before the optimization process ensures that inflation costs can be ignored and excluded from the model. The explanation is based on the assumption that lacking denominations which are not part of a boundary will not cause inflation because there still will be upper and lower representatives with the same types of monetary items (coins or banknotes). According to the assumption mentioned above, the variation in denominations inside the structure with the given boundary will make all scenarios of banknote denomination structures face the same inflation cost (if there is one). The comparison among scenarios in order to obtain the best one can therefore ignore that same amount of inflation costs.

[13] The boundary of currency denomination structure means the borders of coin and banknote series, which includes the lowest-highest denomination value and the transition between coins and banknotes. It is controlled and given by D-Metric and some experts' judgments. Details on the denomination boundary are discussed in chapter 4.

3.4 Cash Payment Profile

This component shows the distribution of cash payments, which highlights the features of cash payment in society, for example, the mode, mean, median, maximum, minimum, and range, especially when there are some changes in the economic situation, such as e-payment, GDP, and inflation. Moreover, it implies a price setting pattern, e.g. distribution of last digit number and the steps in increasing and decreasing prices.

The cash payment profile can be obtained using the survey method. Payers would be asked for their cash payment amounts for goods and services, including tips and donations. There will be various figures according to the types of goods, and places where the goods are purchased, and payers' information, e.g. gender, career, and age. The profile is expected to begin with the minimum cash payment paid via a few of coins and end with the maximum cash payment paid via numerous banknotes (possibly with coins). The gap between consecutive payment amounts should be equal to the average pricing step if the survey is thoroughly conducted. The frequencies of all payment amounts are collected and plotted on a histogram in order to obtain a cash payment profile. There are three main points to be discussed about cash the payment profile: the characteristics of cash payments, the factors affecting a cash payment profile, and the weighted average for the cash payment efficiency analysis.

3.4.1 The Characteristics of Cash Payment Determined from its Profile

Considering the boundary of a profile, the estimated upper and lower bounds represent maximum and minimum cash payment amounts, respectively. The range of the cash payment and the number of payment amounts indicate the gap between adjacent amounts. The greater the number of payment amounts, the narrower the gap will be. This indirectly expresses the lowest unit of coins usually used for transaction purposes, which implies the pricing step that merchants consider for adjusting prices up or down.

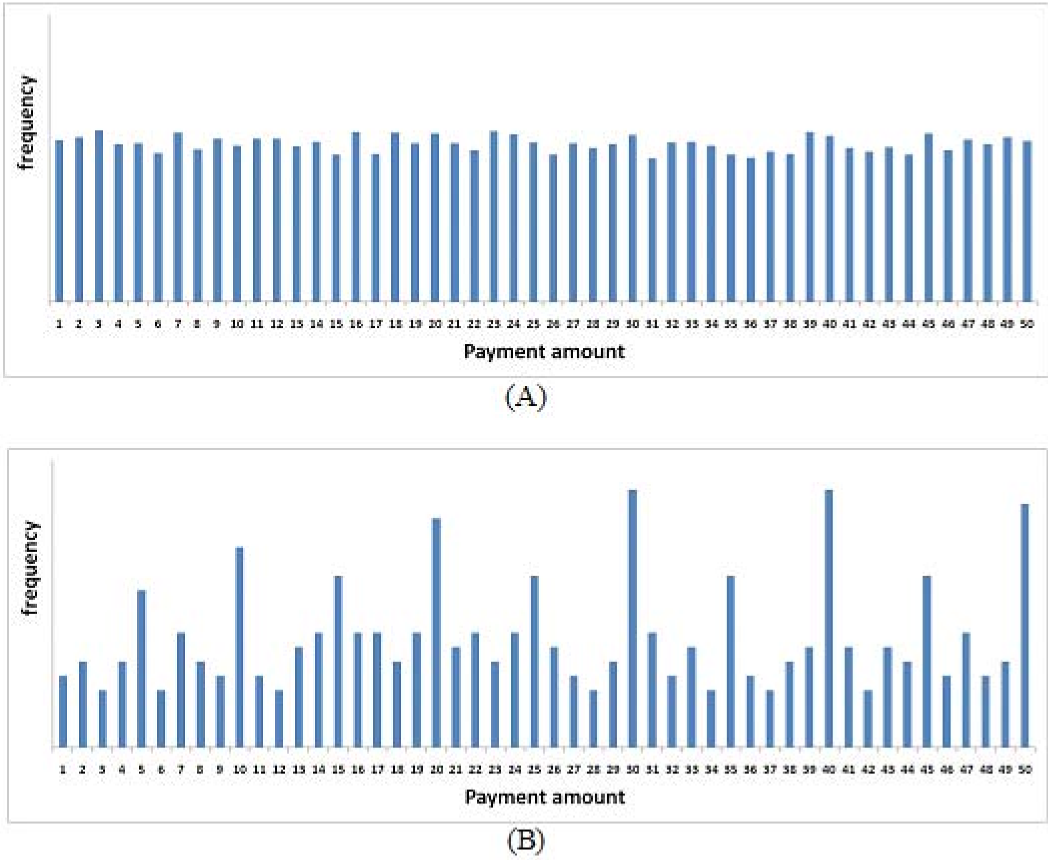
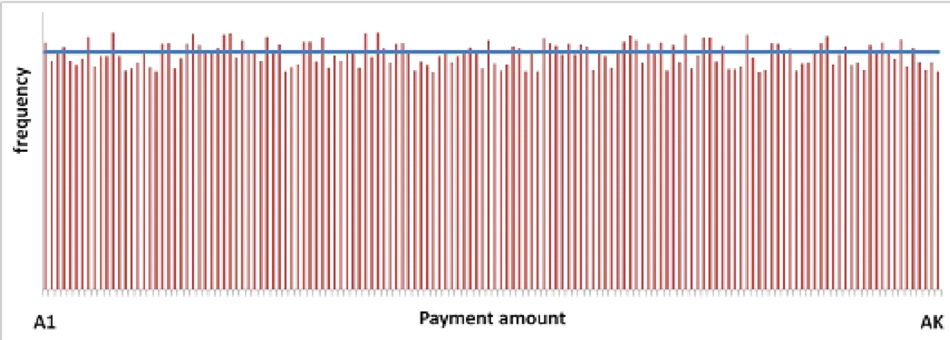
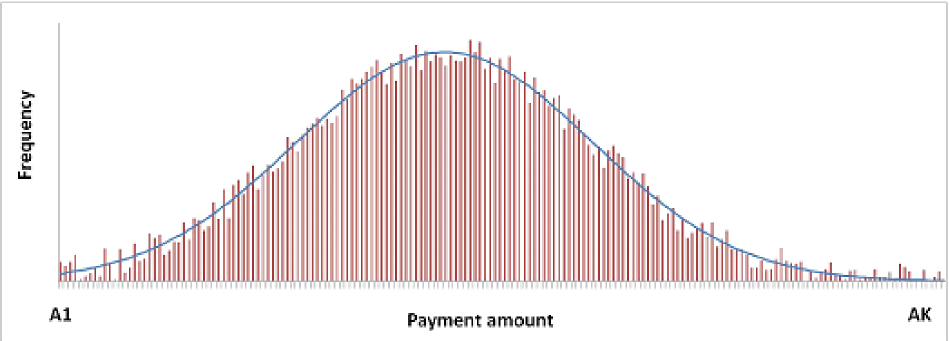


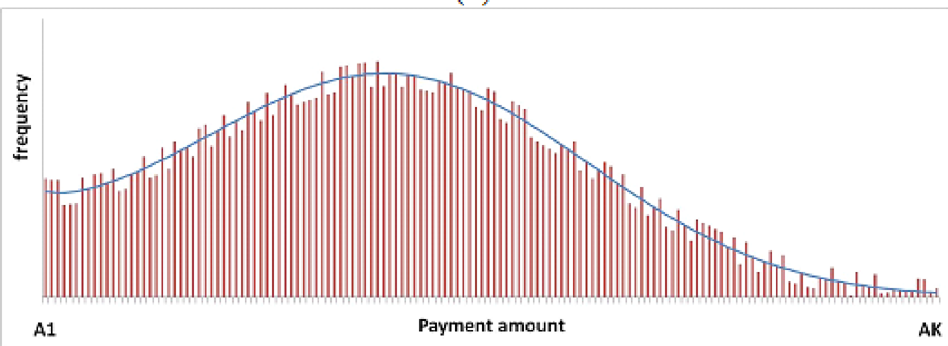
Figure 3.3 Cash Payment Distribution (Short-Series Profile)



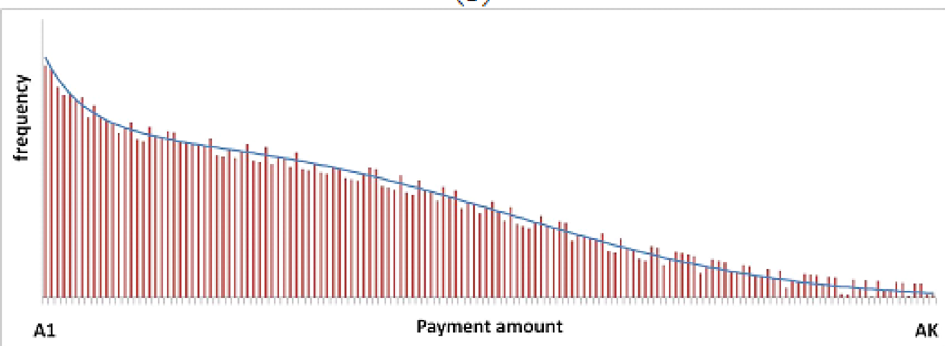
(A)



(B)



(C)



(D)

Figure 3.4 Cash Payment Distribution (Long-Series Profile)

Figure 3.3 explains the fluctuation of cash payments by considering a short-series profile with payment amounts stepped by 1. Panel A shows the random payment and it can be assumed that the all payments have equal possibilities of occurring, whereas panel B shows the cyclical pattern to favor payments with '0' and '5' to be the last digits. The explanation of the latter case is that prices end by '0' the most, followed by '5'and other numbers, respectively. Therefore, the profile is composed of several peaks at the payments amount ending with'0' and '5,' as shown in panel B. However, there may be some arguments where, actually, the payment amounts are generally settled from the combination of goods, such as food and water, books, and pens. In the case where retail shops usually set up the price with '9' being the last digit for psychological purposes, the last digit of the payment amount can therefore be random, depending on how many pieces the payer buys. For example, if he/she buys 3 pieces, the payment amount will end with '7.' Nevertheless, if the most of the goods' prices end with '5' or '10,' the prices of the combinations of those goods will still end with '5' or '10.' Hence, the probability of payments amount ending with'0' and '5'will be higher than in other cases.

The distribution of cash payments can be observed from the whole profile, which is a long series of cash payments. With a long-series profile, we are interested in the whole trend rather than the fluctuation inside a profile. Figure 3.4 shows examples of cash payment profiles in various patterns.

Panel A shows the case of uniform distribution. It can be concluded from this profile that the cash payments have equal possibility of occurring. It is a simple case but is the least realistic because in real situations, there should be differences among payment amount possibilities.

Panel B shows the case of normal distribution, where it can be explained that people in society make payments gathering around one payment amount equal to the mode, mean, and median of all payment amounts. For example, people usually make payments at around 400-500 baht and other payment amounts occur with the fewer possibilities and symmetry of normal distribution.

Panel C shows the case of skewed distribution. This might stem from credit card usage in terms of pushing the peak of the distribution curve to the smaller payment amount if we believe that credit cards or other e-payments play a major role in large payment amounts rather than small payment amounts. For example, people usually make payments via credit cards if the payment amount is more than 500 baht, whereas they pay via cash for payment amount below 500 baht. It will be either the decision of the shops to allow the use of credit cards or the convenience of buyers to use cash without waiting for the authorizing process.

Panel D shows the case of the downward sloping profile, where it can be explained that the bigger the payment amount is, there is less possibility of the payment amount occurring. This will come from the extreme belief that e-payments play a major role in large payment amounts, while cash still plays a major role in small payment amounts. Moreover, smaller payment amounts might occur more often than larger payment amounts. For example, small payment amounts for a bus fee occur two times a day while large payment amounts for shopping at a department store occur perhaps two times a week. This would support the case of higher frequency for low payment amounts.

However, besides credit cards, there is e-money and smart purse, which can pay for small amounts instead of cash. Moreover, some people may not able to use credit cards due to income constraints. These two points present arguments for the profile in panel C and D.

It can be seen that there are still several cases besides the four patterns shown in Figure 3.6, such as the profile with more than one peak. There have been some debates regarding cash payment profile in both short- and long-series determinations. Different societies or countries may generate different profiles according to sellers and buyers' behaviors, e.g. the pricing of merchants, buying manners, etc. The best way to clarify these debates is with direct surveys through thorough interviews or data collection from various places such as retail shops, department stores, hospitals, etc. The specific profile we obtain, therefore, will more accurately represent the cash payment pattern in our society. The obtained result may indicate that there is no exact pattern to be formed or identified by any distribution function.

3.4.2 The Factors Affecting a Cash Payment Profile

The cash payment profile can change over time according to the following factors: consumers' preference and behaviors, e-payments, inflation, price level, income which can be proxied by GDP, and other macroeconomic variables. The impact of those factors on the cash payment profile may change the distribution shape, the location and the gap between adjacent payment amounts. For example, as price level goes up, the step of pricing may increase from 1 to 5 baht. At the same time, the upper and lower boundary would shift to the right due to the higher prices, while e-payments may not affect the payment amount boundary. Nevertheless, if we believe that e-payments as credit cards favor large payment amounts, the distribution may be skewed by moving the peak to the left; in other words, the average cash payment amount decreases.

With the cash payment profile we obtain, payment amounts can be simulated by determining the boundary of a profile and the step between adjacent payment amounts. This simulation process would be used for the cash payment efficiency analysis, which will be discussed later.

Let A_k be the cash payment amount; k = 1, 2, 3, …, K for K payment amounts. A_1 and A_K are defined to be the lower and upper bounds of the profile, while the gap between A_{k-1} and A_k is specified according to the profile domain and the determination of pricing step or the average unit of pricing to be adjusted up (or down).

3.4.3 The Weighted Average for Cash Payment Efficiency Analysis

For the case of the non-uniform cash payment profile, especially for a cyclical pattern, the cash payment efficiency analysis should be carried out using the weighted average of the efficiency calculated from each payment amount given a specific denomination structure. The frequency of each payment amount is therefore used to be a weigher.

Let

A_k be the cash payment amount; k = 1, 2, 3, …, K

f_k is the frequency of cash payment amount A_k

w_k is the weigher of cash payment amount A_k

With the cash payment profile, we can calculate weight (w_k) for payment amount A_k for k = 1, 2, 3, …, K as follows:

$$w_k = \frac{f_k}{\sum_{k=1}^{K} f_k} \quad \text{where } \sum_{k=1}^{K} w_k = 1 \qquad (3.9)$$

3.5 Cash Payment Efficiency

Efficient cash payments are in the interest of the public, retailers, commercial banks, and the central bank. It seems that the costs of counting, controlling, transportation, security, sorting, and destruction will be reduced if people make payments more efficiently. The most widely-used definition of efficient cash payment follows "the principle of least effort," which concerns the average number of coins and banknotes involved to settle the payment. If individuals behaved according to this principle, each amount would be paid such that the number of notes and coins exchanged would be minimized among all possible combinations of the amount of coins and banknotes needed to realize this cash transaction. Such payment schemes are considered to be efficient payments; however, each payment amount may have one or more efficient payment schemes. Taking the example of the 5-denomination structure (1, 2, 5, 10, 20), the payment amount 15 can be efficiently paid in 2 different ways, that is, 10 + 5 and 20 + 5 returned.[14]

Cramer (1983: 299-300) constructed an algorithm to generate all efficient payment schemes for a given range of amounts. According to Cramer, efficient payment can be defined in the same way as mentioned above, i.e. the payment of an arbitrary amount in which the number of banknotes and coins exchanged is minimized. Cramer's model also provides a simple way to illustrate basic differences between denomination ranges. Kippers, Van Nierop, Paap and Franses (2003: 484-

[14] Note that 5 + 5 + 5 is not efficient because the number of tokens used is three, which is not minimized.

508) empirically studied cash payment in the Netherlands using Cramer's algorithm to generate efficient payment schemes. They applied Cramer's algorithm to compare two denomination structures, Euro versus Dutch Guilder, concerning which one was more efficient. In order to compare the efficient payment schemes for different currency ranges, they proposed two aspects of efficiency to be distinguished. First, one can adopt the notion that the smaller the number of tokens exchanged on average, the more efficient it is. Second, they assumed that the more that efficient payment schemes exist for a certain amount, the more opportunities individuals have to make an efficient payment. Put in other words, they assumed that the more efficient payment schemes there are with a small number of tokens, the higher the efficiency of the range is. In this study, these two aspects are therefore used to measure cash payment efficiency according to the principle of least effort.

In this study, average number of minimum tokens and average number of efficient payment schemes are applied to represent the cash payment efficiency in two aspects. Their concepts and mathematical explanation can be explained as follows.

3.5.1 Average Number of Minimum Tokens

The average number of minimum tokens represents cash payment efficiency in the sense that payers and payees prefer making a payment with few monetary items, including change or money returned, for convenience. Therefore, the structure with a lower average number of minimum tokens would generate higher payment efficiency. This is in line with the principle of least effort.

3.5.2 Average Number of Payment Schemes

The average number of payment schemes represents cash payment efficiency in the sense that the structure which has more schemes to make efficient payments would generate higher payment efficiency. This is an extension of the principle of least effort, which focuses on the number of minimum tokens used for payments.

3.5.3 Mathematical Explanation

Let

D = full series denomination structure composed of I denominations

= the system of D_i ; i = 1, 2, 3, …, I

= $[D_1 \quad D_2 \quad D_3 \quad \dots \quad D_I]$

where D is scoped and given at the beginning of the analysis. It is fixed and does not change over time.

X = the dummy used to identify the banknote denomination structure according to D

= $[x_1 \quad x_2 \quad x_3 \quad \dots \quad x_I]$

where $x_i = \begin{cases} 1 \text{ if } \mathrm{D_i} \text{ does exist in the structure} \\ 0 \text{ if } \mathrm{D_i} \text{ does not exist in the structure} \end{cases}$; i = 1, 2, 3, …, I

A_k = payment amounts used for efficiency determination; k = 1, 2, 3, …, K

$n_i(A_k|\mathrm{X})$ = the number of monetary items[15] with denomination D_i for payment amount A_k when denomination structure X is given.

$N(A_k|\mathrm{X})$ = the system of $n_i(A_k|\mathrm{X})$; i = 1, 2, 3, …, I

= $[n_1(A_k|\mathrm{X}) \quad n_2(A_k|\mathrm{X}) \quad n_3(A_k|\mathrm{X}) \quad \dots \quad n_I(A_k|\mathrm{X})]$

tok(X) = Average number of minimum tokens for denomination structure X

sch(X) = Average number of efficient payment schemes for denomination structure X

For one payment amount A_k with specific currency denomination structure X, the following problem is set up for payer and payee to make an efficient payment according to the principle of least effort.

[15] Note that the value can be a positive, zero, or negative number (the negative sign denotes money returned from a merchant).

Minimize total number of tokens = $\sum_{i=1}^{I}|n_i(A_k|\text{X})|$

s.t. $\sum_{i=1}^{I} n_i(A_k|\text{X}) \cdot x_i \cdot D_i = A_k$

let $N^*(A_k|\text{X})$ be the solution, which is the system of $n_i^*(A_k|\text{X})$; i = 1, 2, 3, …, I and there can be more than one solution.

The objective function explains the total number of tokens used for payment amount $A_{k,}$ including money returned from merchants, whereas the constraint controls the total value of all tokens to be equaled to A_k.

According to the solutions for payment amount A_k from given denomination structure X, two figures are considered:

1) the minimum number of tokens used for payment amount A_k = $\sum_{i=1}^{I}|n_i^*(A_k|\text{X})|$, which is denoted as $T_k(A_k|\text{X})$

2) the number of solutions or schemes which yield efficient payments for payment amount A_k, which is assumed to be $S_k(A_k|\text{X})$

(The numerical examples are shown in chapter 4.)

To determine cash payment efficiency, the cash payment amount A_k is simulated, where k = 1, 2, 3, …, K and the weighted average are applied according to the cash payment profile.

The cash payment profile shows the distribution of the cash payment. It can be seen from the profile what the range of payment would be, and whether there are some peaks or some patterns, e.g. the payments always ending with 0 and 5, and payments having a peak at around 500 -1000 baht. With the cash payment profile, we can calculate the weight (w_k) for payment amount A_k for k = 1, 2, 3, …, K, where $\sum_{k=1}^{K} w_k$= 1.

Two aspects of cash payment efficiency are obtained:

1) the average minimum tokens for the denomination structure X is

$$\text{tok(X)} = \sum_{k=1}^{K} T_k(A_k|\text{X}) \cdot w_k \quad (3.10)$$

2) the average number of efficient payment schemes for the denomination structure X is

$$\text{sch(X)} = \sum_{k=1}^{K} S_k(A_k|\text{X}) \cdot w_k \qquad (3.11)$$

It can be seen that the cash payment efficiency analysis in both aspects, which are the average minimum tokens and the average number of efficient payment schemes, depends on:

1) Denomination structure (X)

Different structures would generate their own cash payment efficiency in both aspects.

2) Cash payment profile, which implies the two following aspects:

(1) Payment amount (A_k) ; k = 1, 2, 3, …, K

The set of A_k should be determined from the cash payment profile. The maximum and minimum payment amounts should come from the upper and lower bounds in a profile. The step or difference between the two consecutives A_k and A_{k+1} should come from the frequency of the cash payment profile. The maximum-minimum payment amounts and the step are set to be a boundary of payment range denoted by "start-step-end" to represent the beginning value, the step between consecutive payments, and the end value, respectively. In sum, the simulation of the cash payment amount should be done together with the observation of the cash payment profile.

(2) Weight (w_k)

As discussed earlier, w_k can be obtained from the cash payment profile.

In order to observe the "ins and outs" of cash payment efficiency calculation, some specific experiments, simulations, and examples are shown in chapter 4.

3.6 Optimization Problem According to Cost and Cash Payment Efficiency Perspectives

According to the conceptual framework and the model shown earlier, it can be seen that this research concerns both supply and demand sides and attempts to find the optimal structure in terms of minimum cost and maximum utility with high payment efficiency.

The model also shows the use of the microeconomic foundation to solve the large-scale problem, which is the optimal denomination structure for a country. The procedure begins with utility maximization, which is the basis of the demand analysis. Due to the variety of consumers, which herein stand for the people in society, the characteristic model is selected in this research to estimate the demand share for banknotes by denomination. With the characteristic model, the historical aggregate data and some money features are used to solve the microeconomic problem, which is the demand function. It can be said that the research combines the macroeconomic and microeconomic perspective in a model. Moreover, combining knowledge from various sciences, such as demand analysis and the economic impacts from economics, the cost and efficiency from operation research would provide a wider perspective for the research and make the results more compatible with all economic agents.

As a final point, the optimization problem will be examined according to two themes: static and dynamic analyses with different purposes. Static analysis is evaluated by looking at the situation at a specific time and as to whether the denomination structure is optimal. On the other hand, the dynamic analysis is determined for planning purposes since the re-structure of the currency denomination takes a long time to be applied on a country scale and therefore requires long-term scheduling. The details of the model for each theme are shown as follows:

Let

A = vector of $A_{ji,}$ which contains J characteristics and I denomination

$$= \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1I} \\ A_{21} & A_{22} & \cdots & A_{2I} \\ \vdots & \vdots & \ddots & \vdots \\ A_{J1} & A_{J2} & \cdots & A_{JI} \end{bmatrix}$$

where A_{ji} is an individual (specific) characteristic j for banknote denomination i; j = 1,2,3, ..., J and i = 1,2,3, ..., I

B = vector of $B_{k,}$ which contains K characteristics

$$= \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ \vdots \\ B_K \end{bmatrix}$$

where B_k is common characteristic k for all banknote denominations; k = 1,2,3, ..., K

D = Full series denomination structure composed of I denominations, which are $D_1, D_2, D_3, \ldots, D_I$.

$$= [D_1 \quad D_2 \quad D_3 \quad \ldots \quad D_I]$$

where D is scoped and given at the beginning of the analysis. It is fixed and does not change over time.

X = the dummy used to identify the banknote denomination structure according to D

$$= [x_1 \quad x_2 \quad x_3 \quad \ldots \quad x_I]$$

where $x_i = \begin{cases} 1 \text{ if } \mathrm{D_i} \text{ does exist in the structure} \\ 0 \text{ if } \mathrm{D_i} \text{ does not exist in the structure} \end{cases}$

F = the existent banknote denomination structure according to D

$$= [F_1 \quad F_2 \quad F_3 \quad \ldots \quad F_I] = [D_1 x_1 \quad D_2 x_2 \quad D_3 x_3 \quad \ldots \quad D_I x_I]$$

The denomination structure can be interpreted in the following way:

The elements in F express the denomination values. The positive number of F_i shows the existent i^{th} denominations (D_i), while the zero number of F_j implies that there is no j^{th} denomination (D_j) in the structure.

e.g. in case of the full structures within the range of 1 to 50 according to 1-2-5 series

$$D = [D_1 \quad D_2 \quad D_3 \quad D_4 \quad D_5 \quad D_6] \ ; \ I = 6$$
$$= [1 \quad 2 \quad 5 \quad 10 \quad 20 \quad 50]$$

and for the structures with 4 denominations, which are 1, 5,10 and 20

$$X = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]$$
$$= [1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0]$$

therefore

$$F = [F_1 \quad F_2 \quad F_3 \quad F_4 \quad F_5 \quad F_6]$$
$$= [1 \quad 0 \quad 5 \quad 10 \quad 20 \quad 0]$$

It can be implied from F that there are 4 banknote denominations (positive elements), which are 1, 5, 10 and 20.

Other notations:

$TC_i(\bullet)$ = total cost function of denomination i

$BP_i(\bullet)$ = banknote production planning function of denomination i

$NIC_i(\bullet)$ = banknote in circulation function of denomination i

$S_i(\bullet)$ = banknote demand share function of denomination

M = total banknote supply

L_i = lifespan of banknote with denomination i

X_{-1} = 1-period lag of X

= $[x_{1(-1)} \quad x_{2(-1)} \quad x_{3(-1)} \quad \cdots \quad x_{I(-1)}]$

VC_i = average variable cost (unit cost) of denomination i

FC_{1i} = general fixed cost (from routine process) of denomination i

FC_{2i} = fixed cost from introduction of denomination i

FC_{3i} = fixed cost from elimination of denomination i

The total cost function of banknote denomination i (TC_i) can be implicitly set up as follows:

$$\boldsymbol{TC_i(\bullet) = TC_i(BP_i(NIC_i\,(S_i(X, F(D, X);\ A, B); M), L_i); X_{-1}, VC_i, FC_{1i}, FC_{2i}, FC_{3i})}$$

To explicitly determine the total cost of denomination i, four components are required:

1) Banknote demand share by denomination (S_i)

According to characteristic model and equation (3.5),

$$\boldsymbol{S_i(X, F(D, X);\ A, B)} = \frac{(e^{\sum_j \alpha_j A_{ji} + \sum_k \beta_k B_k F_i}) \cdot F_i^{1-\sigma}}{\sum_{i=1}^{I} [(e^{\sum_j \alpha_j A_{ji} + \sum_k \beta_k B_k F_i}) \cdot F_i^{1-\sigma}]} \tag{3.12}$$

2) Amount of banknotes in circulation (no. of notes) by denomination (NIC_i)

With total banknote supply (M) and banknote demand share by denomination (S_i), together with equilibrium condition, i.e. banknote supply = banknote demand,

$$\boldsymbol{NIC_i\,(S_i; M)} = \frac{S_i \cdot M}{D_i} \tag{3.13}$$

3) Banknote production plan by denomination (BP_i)

With banknotes in circulation (NIC_i) and life of banknotes by denomination (L_i),

$$\boldsymbol{BP_i\,(NIC_i; L_i)} = \frac{12NIC_i}{L_i} \tag{3.14}$$

4) Total cost function (TC_i)

The total cost structure was earlier discussed in section 3.3. In brief, total direct cost can be mainly divided into total variable and total fixed costs:

(1) Total variable cost

with average variable cost (unit cost) of denomination #i (VC_i) and banknote production plan by denomination (BP_i),

$$\text{Total variable cost for denomination i} = BP_i \cdot VC_i \tag{3.15}$$

(2) Total fixed cost

To formulate total fixed cost function, the fixed cost component will be subdivided into three groups:

i) 1st group

The 1st group is composed of the fixed costs that are irrelevant to the introduction and elimination of some banknote denominations. They are defined in section 3.3 as general (from routine process) fixed costs. The examples of the 1st group of fixed costs are costs for machine set up and maintenance costs. On the other hand, examples of the fixed costs excluded from this group are issuing costs, announcing costs, design costs, and costs for ATM modification which are not routinely paid but only incurred from currency re-denomination. Total 1st group fixed cost are assumed to vary by denomination and denoted by FC_{1i} for denomination i. Therefore, total 1st group fixed cost for denomination i = FC_{1i}

It can be seen that the overall (all denominations) fixed cost in 1st group directly relates to the number of denominations. The greater the number of denominations, the higher the total amount of 1st group fixed costs that will be incurred.

ii) 2nd group

The 2nd group is composed of fixed costs from the introduction of new denominations, such as issuing and announcing costs. The 2nd group fixed costs will not be incurred unless the denomination structure introduces new denominations. For the introduced denominations, the 2nd group fixed costs are assumed to vary by denomination and are denoted by FC_{2i} for denomination i. It can be seen that the 2nd group of fixed costs depends on the banknote denomination structure dummy of this period (X) and last period (X_{-1}); that is, whether there are any denominations introduced in the structure. Therefore, total 2nd group fixed cost for denomination i = $FC_{2i} \cdot x_i \cdot (x_i - x_{i(-1)})$

iii) 3rd group

The 3rd group is composed of fixed costs from the elimination of old denominations such as announcing costs. The 3rd group of fixed costs will not be incurred unless the denomination structure gets rid of some denominations. For the eliminated denominations, the 3rd group fixed costs are assumed to vary by denomination and are denoted by FC_{3i} for denomination i. It can be seen that the 3rd group of fixed costs depends on the banknote denomination structure of this period (X) and the last period (X_{-1})—whether there are any denominations eliminated from the structure. Therefore, total 3rd group fixed cost for denomination i = $FC_{3i} \cdot x_{i(-1)} \cdot (x_{i(-1)} - x_i)$

According to i) – iii), the total fixed costs for denomination i can be formulated as (3.16):

$$\begin{aligned}&\text{Total fixed costs for denomination i}\\&= FC_{1i} + FC_{2i} \cdot x_i \cdot (x_i - x_{i(-1)}) + FC_{3i} \cdot x_{i(-1)} \cdot (x_{i(-1)} - x_i)\\&= FC_{1i} + FC_{2i} \cdot {x_i}^2 + FC_{3i} \cdot x_{i(-1)}^2 - (FC_{2i} + FC_{3i}) \cdot x_i \cdot x_{i(-1)} \qquad (3.16)\end{aligned}$$

Referring to (3.15) and (3.16), the total costs for denomination i can be formulated as follows:

$$\begin{aligned}&\boldsymbol{TC_i(BP_i; VC_i, FC_{1i}, FC_{2i}, FC_{3i})}\\&= BP_i \cdot VC_i + FC_{1i} + FC_{2i} \cdot {x_i}^2 + FC_{3i} \cdot x_{i(-1)}^2 - (FC_{2i} + FC_{3i}) \cdot x_i \cdot x_{i(-1)}\\&= \frac{12S_i \cdot M \cdot VC_i}{L_i \cdot D_i} + FC_{1i} + FC_{2i} \cdot {x_i}^2 + FC_{3i} \cdot x_{i(-1)}^2 - (FC_{2i} + FC_{3i}) \cdot x_i \cdot x_{i(-1)}\\&\text{where } S_i = \frac{(e^{\sum_j \alpha_j A_{ji} + \sum_k \beta_k B_k F_i}) \cdot F_i^{1-\sigma}}{\sum_{i=1}^{I} [(e^{\sum_j \alpha_j A_{ji} + \sum_k \beta_k B_k F_i}) \cdot F_i^{1-\sigma}]} \qquad (3.17)\end{aligned}$$

Therefore the "static problem" can be shown as (3.18).

Minimize total cost =

$$\sum_{i=1}^{I}\left[\frac{12(e^{\sum_j \alpha_j A_{ji}+\sum_k \beta_k B_k F_i})\cdot F_i^{1-\sigma}\cdot M\cdot VC_i}{L_i\cdot D_i\cdot \sum_{i=1}^{I}[(e^{\sum_j \alpha_j A_{ji}+\sum_k \beta_k B_k F_i})\cdot F_i^{1-\sigma}]}\right] + FC_{1i} + FC_{2\mathrm{i}}\cdot {x_i}^2 + FC_{3\mathrm{i}}\cdot x_{i(-1)}^2 - (FC_{2\mathrm{i}} + FC_{3\mathrm{i}})\cdot x_i\cdot x_{i(-1)} \quad (3.18)$$

Choice variable: X, which is composed of $x_1, x_2, x_3, \ldots, x_I$

Subject to

1) Cash supply = Cash demand (in equilibrium)
2) The denomination structure follows binary-decimal triplets (1-2-5-10-20-50)
3) Given boundary from D-Metric
   (1) Lowest coin denomination
   (2) Transition between coins and banknotes which is identified by lowest banknote denomination
   (3) Highest banknote denomination
4) Cash payment efficiency

(1) Two aspects according to the principle of least effort are controlled, average minimum tokens denoted by tok(X) and average payment schemes which would give efficient payment denoted by sch(X).

(2) The criteria depend on policy makers in terms of the extent to which they are concerned about cash payment efficiency. For example, the cash payment efficiency must not be worse than the value given by a current denomination structure.

5) Full series denomination structure (D)

In this research, concerning the case of Thailand, the full series are given and fixed as follows:

$$D = [10 \quad 20 \quad 50 \quad 100 \quad 200 \quad 500 \quad 1000 \quad 2000 \quad 5000]$$

In Thailand, the current (at 2011) banknote denomination dummy (X) is

$$X = [0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0]$$

Hence,

$$F = [0 \quad 20 \quad 50 \quad 100 \quad 0 \quad 500 \quad 1000 \quad 0 \quad 0]$$

It can be seen from F that there are five banknote denominations, which are 20, 50, 100, 500 and 1000 baht.

6) Other assumptions (for numerical analysis), for example

(1) Denominations based on 10^n are always in the series

(2) Denominations based on $2*10^n$ and $5*10^n$ are not allowed to be skipped in each loop with the same n.

3.6.1 The Graphical Explanation

The optimization problem in this study regards the cost and efficiency obtained by the specific denomination structure. According to Bouhdaoui, Bounie, and Hove (2011: 20), cost and payment efficiency might be a trade-off each other depending on the fixed and variable costs, whether which one dominates the other. In general, the costs related to currency issuing, including production, storing, transportation, and destruction, can be mainly divided into fixed and variable costs, as with other costs in the industry. The variable costs positively relate to the number of monetary items in circulation, which implies the tokens to be produced and distributed. On the other hand, the number of denominations would increase fixed costs, which are separately incurred by denomination and defined in this study as routine process fixed costs, e.g. maintenance and storage costs. If we believe that the various types of denominations would support the payment efficiency in the sense of minimum tokens paid for each transaction, the higher number of denominations then reduces the variable costs due to the lower number of monetary items demanded by the public and supplied by producers. As a result, fixed and variable costs are a trade-off of each other according to the movement of the number of denominations in the structure, which directly relates to cash payment efficiency. In sum, the higher the number of denominations there is in a structure, the higher the fixed costs that will be incurred, but the fewer monetary items from higher cash payment efficiency there will be in circulation, which would generate lower variable costs. As discussed earlier, payment efficiency and total cost may or may not be a trade-off of each other, depending on the comparison between the fixed and variable costs and also on how well the number of denominations reflects the number of monetary items in circulation.

To express the story above via a mathematical model, cost and payment efficiency have to be clearly defined. According to equation (3.18), the total cost incurred from all denomination is:

$$\text{Total cost} = \sum_i [\frac{12S_i \cdot M}{L_i \cdot D_i} \cdot VC_i + FC_{1i} + FC_{2i} \cdot {x_i}^2 + FC_{3i} \cdot x_{i(-1)}^2 - (FC_{2i} \ +) \cdot x_i \cdot x_{i(-1)}] \quad (3.19)$$

Assume that the average variable cost and annual general fixed cost per denomination are equal for all denominations, which are denoted by VC and FC_1, respectively. Moreover, the fixed costs of the introduction of a new denomination (FC_2) and old denomination elimination (FC_3) are ignored due to the comparison among various structures in this section in order to examine the movement of the relationship between cost and efficiency according to the various denomination structures. Therefore, equation (3.19) can be simplified as follows:

$$\text{Total cost} = \text{VC} \cdot \sum_i \frac{12S_i \cdot M}{L_i \cdot D_i} + \text{ I} \cdot \text{FC}_1 \quad (3.20)$$

where I is the number of denominations

Note that $\frac{12S_i \cdot M}{L_i \cdot D_i}$ is the number of monetary items with denomination i which is planned for annual production. Therefore $\sum_i \frac{12S_i \cdot M}{L_i \cdot D_i}$ is the total amount of monetary items planned for annual production, denoted in this section by Qm. For simplicity, lifespan (L_i) is assumed to be equal for all denominations and Qm is assumed to be a function of the average number of minimum tokens (tok) i.e. Qm = f(tok). This is logical since the average number of minimum tokens would reveal the currency demand by the public, which reflects the currency in circulation and finally implies the currency production volume. It is simple to see that the relationship between Qm and tok is in a positive direction, i.e. $\frac{\partial \text{f}}{\partial \text{tok}} > 0$ and equation (3.21) would be simplified as:

$$\text{Total cost} = \text{VC} \cdot \text{f(tok)} + \text{ I} \cdot \text{FC}_1 \quad (3.21)$$

It can be seen from total cost function in equation (3.21) that there are two components. The first component is total variable cost denoted by TVC and the other is total fixed cost denoted by TFC. That is:

$$\text{Total cost} = \text{TVC} + \text{TFC} \quad (3.22)$$

$$\text{TVC} = \text{VC} \cdot \text{f(tok)} \quad (3.23)$$

$$\text{TFC} = \text{I} \cdot \text{FC}_1 \quad (3.24)$$

To examine the relationship between cost vs. payment efficiency and also the trade-off effect between fixed cost and variable cost, the first aspect of cash payment efficiency, which is the average number of minimum tokens (tok), is selected to be a proxy[16]. The lower tok value reflects the higher payment efficiency according to the principle of least effort. Moreover, there tends to be a strongly negative relationship between the average number of minimum tokens and number of denominations in a system.[17] It can be logically explained that the larger number of denominations would increase payment efficiency by lowering the number of tokens used in transaction because there are various choices of monetary items to be selected. As a result, it can be implied that tok is a function of the number of denominations (I) and other factors (Xs), if they exist. Hence,

$$\text{tok} = \text{G(I,Xs)} \quad (3.25)$$

where $\frac{\partial G}{\partial I} < 0$.

However, we can also express I in terms of tok by the inverse function of G or G^{-1}. Let function G^{-1} be denoted by g. Thus, we obtain I = g(tok, Xs) and $\frac{\partial g}{\partial tok} < 0$. The fixed cost in equation (3.21) can therefore be expressed in terms of tok as equation (3.26).

$$\text{TFC} = \text{FC}_1 \cdot \text{g(tok, Xs)} \quad (3.26)$$

[16] The second aspect, which is number of efficient schemes (sch), is not considered in this section.

[17] See chapter 4

From equation (3.23) and (3.26), we obtain

$$\frac{\partial \mathrm{TVC}}{\partial \mathrm{tok}} = \mathrm{VC} \cdot \frac{\partial \mathrm{f}}{\partial \mathrm{tok}} \tag{3.27}$$

$$\frac{\partial \mathrm{TFC}}{\partial \mathrm{tok}} = \mathrm{FC}_1 \cdot [\frac{\partial \mathrm{g}}{\partial \mathrm{tok}} + \frac{\partial \mathrm{g}}{\partial \mathrm{Xs}} \cdot \frac{\partial \mathrm{Xs}}{\partial \mathrm{tok}}] \tag{3.28}$$

We first consider the case in which Xs are fixed, implying that $\frac{\partial \mathrm{Xs}}{\partial \mathrm{tok}} = 0$. Hence,

$$\frac{\partial \mathrm{TFC}}{\partial \mathrm{tok}} = \mathrm{FC}_1 \cdot \frac{\partial \mathrm{g}}{\partial \mathrm{tok}} \tag{3.29}$$

As learned earlier, $\frac{\partial \mathrm{f}}{\partial \mathrm{tok}} > 0$ and $\frac{\partial \mathrm{g}}{\partial \mathrm{tok}} < 0$; therefore $\frac{\partial \mathrm{TVC}}{\partial \mathrm{tok}}$ and $\frac{\partial \mathrm{TFC}}{\partial \mathrm{tok}}$ are positive and negative, respectively. This derivation confirms the logic that the higher payment efficiency shown by the smaller average number of minimum tokens (tok) and achieved from a higher number of denominations (I) would lower total variable costs (TVC) but would increase total fixed cost(TFC). It can therefore be implied that there is a trade-off between TVC and TFC. From equation (3.27) and (3.29),

$$\frac{\partial \mathrm{TVC}}{\partial \mathrm{TFC}} = \frac{\mathrm{VC}}{\mathrm{FC}_1} \cdot \frac{\partial \mathrm{f}/\partial \mathrm{tok}}{\partial \mathrm{g}/\partial \mathrm{tok}} \tag{3.30}$$

Equation (3.30) guarantees the trade-off between TVC and TFC due to the positive and negative value of $\frac{\partial \mathrm{f}}{\partial \mathrm{tok}}$ and $\frac{\partial \mathrm{g}}{\partial \mathrm{tok}}$, respectively. The graphical explanation is shown in Figure 3.5.

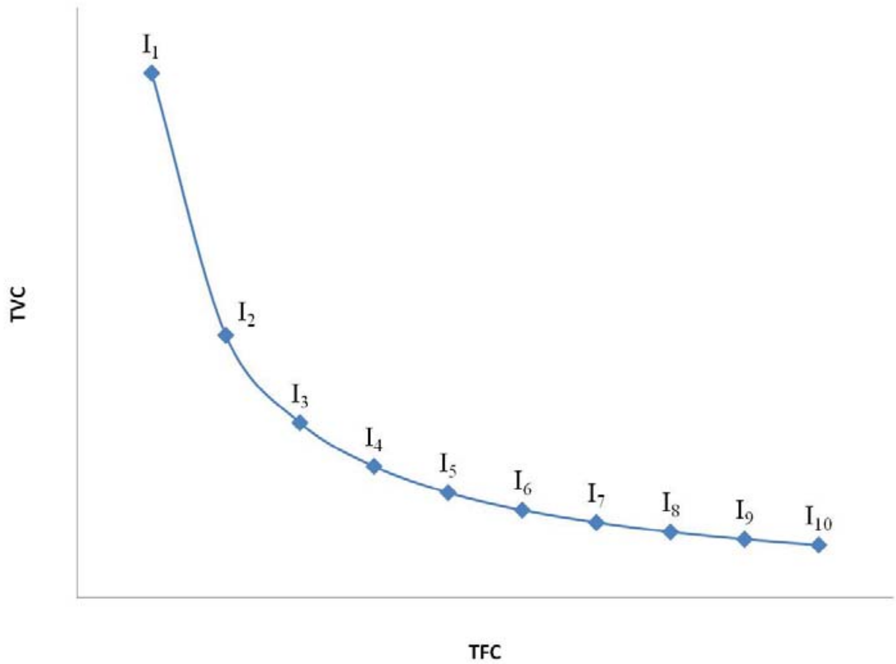


Figure 3.5 Trade-off between Total Fixed Cost and Variable Cost

According to Figure 3.5, the graph shows ten denomination structures, indicated by ten markers. Along the downward sloping trend, a trade-off between TVC and TFC can be observed. The structure with a higher TFC would generate a lower TVC. $I_1 - I_{10}$ denote the structure with the number of denominations equal $I_1 - I_{10}$. Comparing the ten structures, the structure higher to the left would generate a higher TVC but a lower TFC, unlike the structure lower to the right. It can be further implied that $I_1 < I_2 < I_3 < \ldots . < I_{10}$. Moreover the structure with I_1 denominations (least value among ten cases) will gain the lowest payment efficiency with the highest tok value.

From equation (3.22), (3.23) and (3.26), the total cost function would be

$$\text{Total cost} = \mathrm{VC} \cdot \mathrm{f(tok)} + \mathrm{FC_1} \cdot \mathrm{g(tok, Xs)} \tag{3.31}$$

Considering the relationship between cash payment efficiency and total cost,

$$\frac{\partial(\text{total cost})}{\partial \text{tok}} = \frac{\partial \text{TVC}}{\partial \text{tok}} + \frac{\partial \text{TFC}}{\partial \text{tok}}$$

$$= \text{VC}\cdot\frac{\partial \text{f}}{\partial \text{tok}} + \text{FC}_1\cdot\frac{\partial \text{g}}{\partial \text{tok}} \quad (3.32)$$

Again, $\frac{\partial \text{f}}{\partial \text{tok}} > 0$ and $\frac{\partial \text{g}}{\partial \text{tok}} < 0$; therefore $\frac{\partial(\text{total cost})}{\partial \text{tok}}$ might be positive or negative, depending on the average variable cost (VC), general fixed cost per denomination (FC_1), and the marginal effects of tok on Qm and I ($\frac{\partial \text{f}}{\partial \text{tok}}$ and $\frac{\partial \text{g}}{\partial \text{tok}}$).

Considering the graph in the tok-cost plane, as shown in Figure 3.6, we combine TVC, TFC, and total cost in the same figure. From equation (3.27) and (3.29), it can be seen that the TVC and TFC lines have to be upward and downward sloped respectively. However, due to the explicit functions of TVC and TFC being realized, there might be various possible patterns of total cost line, such as upwardly linear, downwardly linear, convex with minimum point, concave with maximum point, downwardly convex, and upwardly concave. From a graphical view, the shape of the total cost line depends on the patterns of the TVC and TFC lines. If they are both assumed to be linear, the total cost line shape has to be linear. Nevertheless, if we believe that the optimal number of denominations (I*) which gives the minimum cost should not be too low or too high value, the non-linear curve with a local minimum point is selected to be a general pattern of total cost function, as shown in the figure. Again the structure with a larger number of denominations will achieve higher efficiency with a lower value of tok and will move toward the left. This supports the idea that $I_1 < I_2 < I_3 < \ldots < I_{10}$.

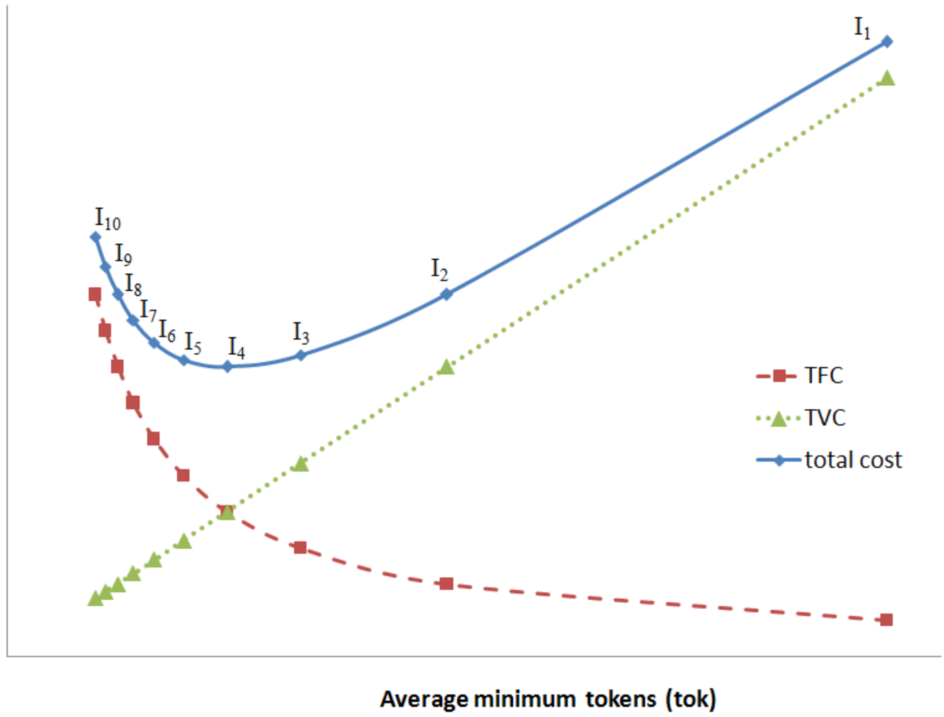


Figure 3.6 The Relationships among tok vs. TFC, TVC, and Total Cost

The total cost curve in Figure 3.6 is pulled out and separately shown again in Figure 3.7. Along the curve, various denomination structures are located. The structure which gives the minimum total cost is the structure at point I_4 (with I_4 denominations). The structures from this point forward to the right give higher total cost and tok value, which explains the lower payment efficiency. Therefore, they are all worse off and dominated by point I_4. On the other hand, the structures from point I_4 backward to the left give a higher total cost but lower tok value, which explains higher payment efficiency. All structures I_5- $I_{10,}$ including $I_{4,}$ therefore face a trade-off condition with a downward sloping trend. Considering the path in the trade-off condition shown as a solid line in Figure 3.7, there are seven structures to be candidates, which are $I_4 – I_{10}$. The structure with the minimum total cost is located the most right in the path (I_4). The structure with the highest payment efficiency is located the most left in the path (I_{10}). Other structures in the path ($I_5 – I_9$) give a higher cost than the structure at $I_{4,}$ but also achieve higher efficiency. They also give a lower cost than the structure at $I_{10,}$ but also achieve lower efficiency. This confirms that the occurrence of a trade-off and the path in the trade-off condition, in which structures $I_4 - I_{10}$ are located, could be said to be an optimal path. However, which structure is optimal cannot be exactly identified unless the weights of the total cost and efficiency are defined by currency authorities or policy makers.

In the mathematical view, the trade-off condition can be considered from the downward slope of the total cost curve, which is explicated by $\frac{\partial(\text{total cost})}{\partial \text{tok}} < 0$. From equation (3.32), therefore, the trade-off condition is

$$\text{VC}\cdot\frac{\partial f}{\partial \text{tok}} + \text{FC}_1\cdot\frac{\partial g}{\partial \text{tok}} < 0 \quad \text{or} \quad \frac{\text{FC}_1}{\text{VC}} > -\frac{\partial f/\partial \text{tok}}{\partial g/\partial \text{tok}} \tag{3.33}$$

(note that $\frac{\partial g}{\partial \text{tok}} < 0$, the inequality sign has to be carefully determined)

It can be shown that $\frac{\partial f/\partial \text{tok}}{\partial g/\partial \text{tok}} = \frac{\partial \text{Qm}}{\partial \text{I}}$, therefore, the trade-off condition would be simplified as

$$\frac{\text{FC}_1}{\text{VC}} > \left|\frac{\partial \text{Qm}}{\partial \text{I}}\right| \tag{3.34}$$

From equation (3.34), it can be explained that a trade-off condition exists if the ratio of general fixed cost by denomination (FC_1) to the average variable cost (VC) is greater than the magnitude of the marginal effect of number of denominations (I) on the total monetary item amount planned for production (Qm).

For the consideration of all possible denomination structures, three cases are raised, as follows:

1) All structures which are located in the interval with $\frac{\text{FC}_1}{\text{VC}} > \left|\frac{\partial \text{Qm}}{\partial \text{I}}\right|$ or a trade-off condition would be realized as optimal structures. The set of them could be called an "optimal path" (curve $I_4 - I_{10}$ for example).

2) In addition to the structures in the interval with $\frac{\text{FC}_1}{\text{VC}} < \left|\frac{\partial \text{Qm}}{\partial \text{I}}\right|$, which is not in the trade-off condition, they would be dominated by the structure located at the most left and in the lowest position (I_4 for example).

3) Moreover, for the U-shaped total cost curve, the condition of $\frac{FC_1}{VC} = \left|\frac{\partial Qm}{\partial I}\right|$ would provide the optimal denomination structure which would give the minimum total cost regardless of the consideration of cash payment efficiency (I_4, for example).

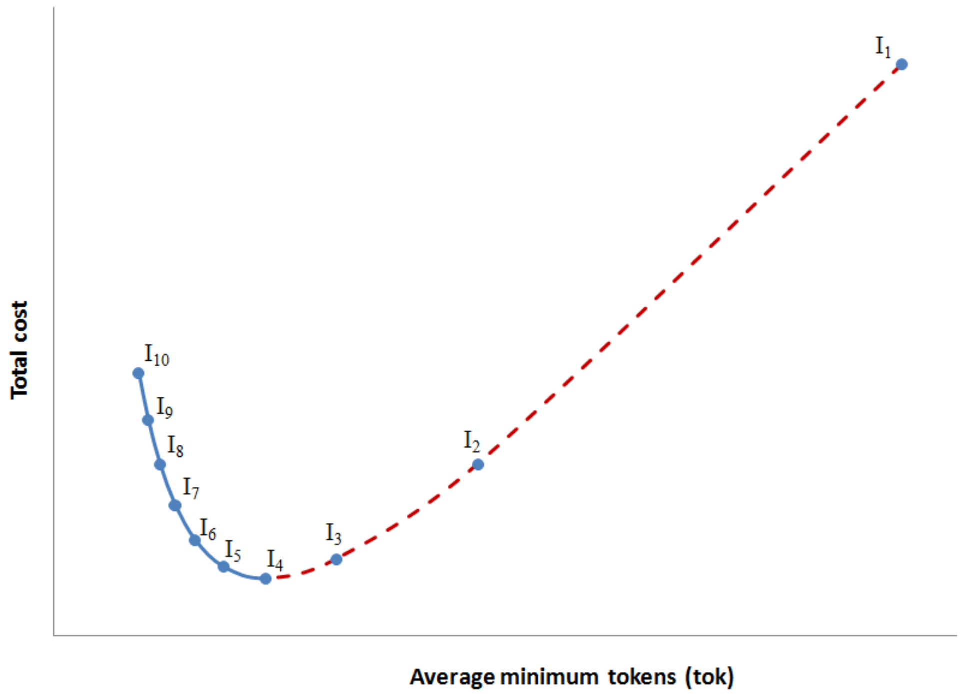


Figure 3.7 Optimal Path of Denomination Structure

According to total cost function as expressed Figure 3.7, the path comes from the determination of the total cost function in equation (3.31) by looking at the slope or $\frac{\partial(\text{total cost})}{\partial \text{tok}}$ from equation (3.32), which implies a ceteris paribus condition since we earlier assumed that Xs were fixed. In other words, the analysis of the shape of total cost function is based on the variations of tok regardless of the impacts from other factors (Xs).

Considering what Xs could be, one of possible Xs is the average space of the denomination structure (SP). For example, the structures 1-2-4-8, 1-2-5-10 and 1-3-9-27, which have the same number of denominations (I=4), do not give equal average number of minimum tokens. The above three structures with average spaces of 2, 2.17

and 3 yield a tok value equal to 4.37, 4.46 and 5.25, respectively[18]. Therefore, the average space should be another factor which helps explain tok. Suppose that the number of denominations (I) and average space (SP) can nearly completely capture the variation of tok[19]. As a result, the Xs in equation (3.25) can be replaced by SP i.e. tok = G(I,SP). The points on the total cost curve in Figure 3.7 represent only the structures with same value of average spaces (SP is fixed as assumed) but different in number of denominations, for example, 1-2-4, 1-2-4-8 and 1-2-4-8-16 (SP = 2 and I = 3, 4, 5, respectively).

This research, then, focuses on the comparisons among the structures with the same boundary but with different details inside. If the boundaries, which are the lowest and highest denominations, are controlled, the number of denominations and average space will always be negatively correlated. Table 3.2 shows examples from the structures which have the same boundary with lowest and highest denomination equal to 1 and 10, respectively. It can be seen that the higher number of denominations there are, the lower the average space of the structure it gets. However, structures with same number of denominations can give either the same or different average spaces, for example, 1-2-10, 1-5-10 and 1-2.5-10.

Table 3.2 The Relationship between Number of Denominations and Average Space among the Structures with the Same Boundary (1-10)

No.	Structure	No. of Denominations	Average Spacing Factor
1	1-2-5-10	4	2.17
2	1-2.5-5-10	4	2.17
3	1-2-10	3	3.50
4	1-5-10	3	3.50
5	1-2.5-10	3	3.25
6	1-10	2	10.0

[18] See section 4.6 in chapter 4.

[19] Number of runs and cash payment boundary are expected to be other factors (see section 4.5 in chapter 4).

In the previous section, we have already realized the cost and efficiency movement along the structure with different numbers of denominations but equal average spaces, according to Figure 3.7. Now we will examine the case of the same number of denominations but with different average spaces, e.g. 1-2-10 and 1-2.5-10, with the average space equal to 3.50 and 3.25, respectively. Logically, the structure with less average space seems to be more efficient according the principle of least effort, implying that the average minimum tokens (tok) is lower. In this case, the change in total cost comes from only the variable cost due to changes in the average space, which affects the tok value. On the other hand, the fixed cost does not change since the number of denominations is controlled and fixed. According to equation (3.21),

$$\Delta \text{Total cost} = \text{VC} \cdot \frac{\partial \text{f}}{\partial \text{tok}} \cdot \Delta\text{tok} + \text{FC}_1 \cdot \Delta \text{I} \qquad (3.35)$$

Since $\frac{\partial f}{\partial tok} > 0$, the lower tok ($\Delta tok < 0$) and constant I ($\Delta I=0$), therefore, leading to a smaller value of total cost. On the other hand, the larger average space would give the structure less efficiency with a higher tok and higher total cost. Figure 3.8 explains the case we latest discussed. In Figure 3.8, the SP denotes the structure with spacing SP. According to the above, it is straightforward to conclude that SP1 < SP2 < SP3 < SP4. It can be further seen that with an equal number of denominations and a controlled boundary, the optimal structure would be the one with the lowest average space.

Figure 3.8 The Relationship between Total Cost and Average Minimum Tokens According to the Variation of Average Space with a Fixed Number of Denominations

In general, if the variations of I and SP are allowed at the same time, there might be some scattered points on the diagrams, as shown in Figure 3.9. The relationship between tok and total cost cannot be represented with one line (or nearly). Looking in depth, we may be able to separate the various scenarios by I and SP, as expressed in Figure 3.10[20]. Again, SP1 – SP9 denote the structure with the average space equal to SP1 – SP9, respectively, I1 < I2 < ….< I9 and SP1 < SP2 < ….< SP9. In order to compare possible scenarios and to select the optimal one, the solutions would be in the lower and left direction. There might be more than one solution on the downward slope path which faces the trade-off condition. However, the cash payment constraints, in which only 1-2-5 structures are allowed, will narrow down the possible

[20] Note that if other factors besides the number of denominations and average space are taken into account, the scattered plot will be more randomly located and it will be more difficult to separate the impacts, as shown in Figure 3.10. For example, the number of runs and payment range (boundary) are expected to affect cash payment efficiency (see chapter 4). Another factor would be social preferences, which have an impact on currency demand and supply by denomination and finally affect the total cost.

candidates by ignoring the dominated structures (lower efficiency and higher cost). Moreover, if the boundary of the denomination is controlled, some scenarios might be excluded and the structures will be less complicated to be compared and selected for optimal solutions.

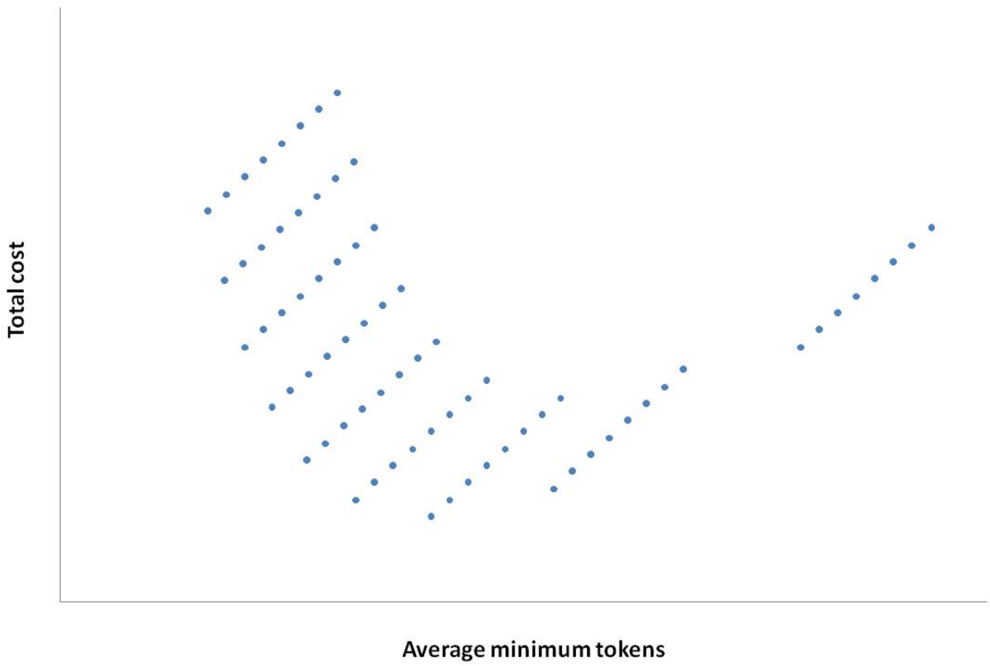


Figure 3.9 General and More Realistic Possible Scenarios

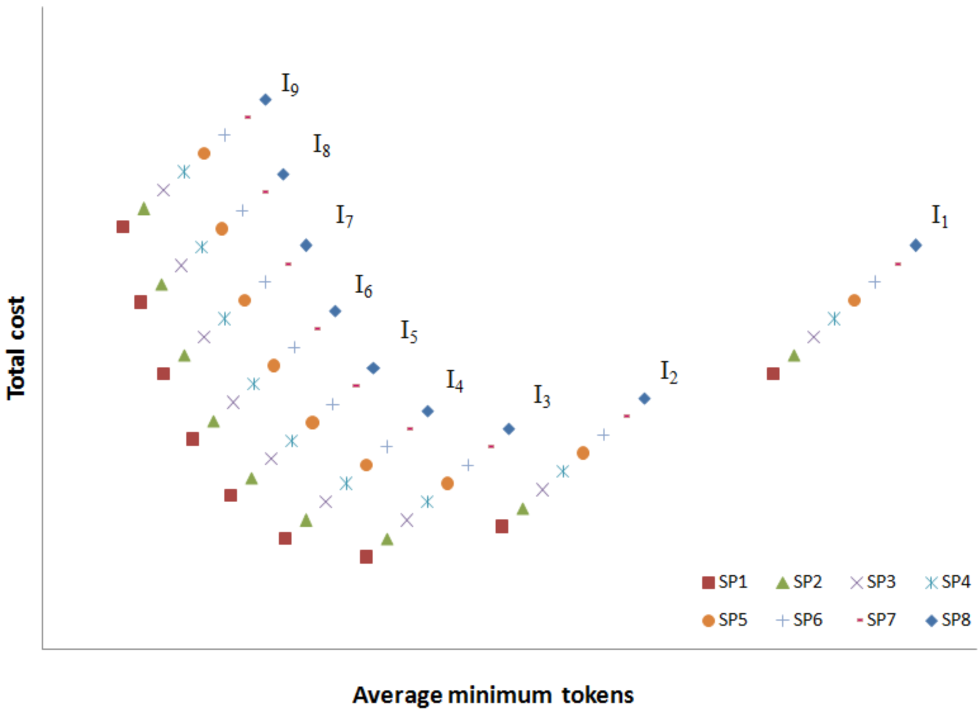


Figure 3.10 Possible Scenarios Separated by Number of Denominations and Average Space

3.6.2 The Extension to Dynamic Analysis

The static problem can be extended to a dynamic analysis, which is used to find the optimal banknote denomination structure according to this research. The "dynamic problem" can be set up as follows:

$$\boldsymbol{TC_i}(\bullet) = \boldsymbol{TC_{it}}(\boldsymbol{BP_{it}}(\boldsymbol{NIC_{it}}(\boldsymbol{S_{it}}(\boldsymbol{X_t}, \boldsymbol{F_t}(\boldsymbol{D}, \boldsymbol{X_t});\ \boldsymbol{A_t}, \boldsymbol{B_t}); \boldsymbol{M_t}), \boldsymbol{L_{it}}); \boldsymbol{X_{t-1}}, \boldsymbol{VC_{it}}, \boldsymbol{FC_{1it}}, \boldsymbol{FC_{2it}}, \boldsymbol{FC_{3it}})$$

where

$TC_{it}(\bullet)$ = cost function of denomination i at period t

$BP_{it}(\bullet)$ = banknote production (plan) function of denomination i at period t

$NIC_{it}(\bullet)$ = banknote in circulation function of denomination i at period t

$S_{it}(\bullet)$ = banknote demand share function of denomination at period t

M_t = total banknote supply at period t

L_{it} = lifespan of banknote with denomination i at period t

A_t = vector of A_{jit} which contains J characteristics and I denominations

$$= \begin{bmatrix} A_{11t} & A_{12t} & \cdots & A_{1It} \\ A_{21t} & A_{22t} & \cdots & A_{2It} \\ \vdots & \vdots & \ddots & \vdots \\ A_{J1t} & A_{J2t} & \cdots & A_{JIt} \end{bmatrix},$$

where A_{jit} is individual characteristic j for banknote denomination i at period t; j = 1,2,3, ..., J and i = 1,2,3, ..., I

B_t = vector of B_{kt} which contains K characteristics

$$= \begin{bmatrix} B_{1t} \\ B_{2t} \\ B_{3t} \\ \vdots \\ B_{Kt} \end{bmatrix}$$

where B_{kt} is common characteristic k for all banknote denominations at period t; k = 1,2,3, ..., K

D = Full series denomination structure (the same as D in static approach)

X_t = the dummy used to identify banknote denomination structure at period t according to D

$= [x_{1,t} \quad x_{2,t} \quad x_{3,t} \quad \cdots \quad x_{I,t}]$

where $x_{i,t} = \begin{cases} 1 \text{ if } D_i \text{ does exist in the structure at period t} \\ 0 \text{ if } D_i \text{ does not exist in the structure at period t} \end{cases}$

F_t = the existent banknote denomination structure according to D

$= [F_1 \quad F_2 \quad F_3 \quad \cdots \quad F_I] = [D_1x_1 \quad D_2x_2 \quad D_3x_3 \quad \cdots \quad D_Ix_I]$

X_{t-1} = 1-period lag of X_t

$= [x_{1,t-1} \quad x_{2,t-1} \quad x_{3,t-1} \quad \cdots \quad x_{I,t-1}]$

VC_{it} = average variable cost (unit cost) of denomination i at time t

FC_{1it} = general fixed cost of denomination i at time t

FC_{2it} = fixed cost from introduction of denomination i at time t

FC_{3it} = fixed cost from elimination of denomination i at time t

Similar to the static model, the total costs for denomination i can be formulated as follows:

$$\boldsymbol{TC_{it}(BP_{it}; VC_{it}, FC_{1it}, FC_{2it}, FC_{3it})}$$
$$= BP_{it} \cdot VC_{it} + FC_{1it} + FC_{2it} \cdot {x_{it}}^2 + FC_{3it} \cdot x_{i,t-1}^2 - (FC_{2it} + FC_{3it}) \cdot x_{i,t} \cdot x_{i,t-1}$$
$$= \frac{12S_{it} \cdot M_t \cdot VC_{it}}{L_{it} \cdot D_i} + FC_{1it} + FC_{2it} \cdot {x_{it}}^2 + FC_{3it} \cdot x_{i,t-1}^2 - (FC_{2it} + FC_{3it}) \cdot x_{i,t} \cdot x_{i,t-1}$$

where $S_{it} = \frac{(e^{\sum_j \alpha_j A_{jit} + \sum_k \beta_k B_{kt} F_{it}}) \cdot F_{it}^{1-\sigma}}{\sum_{i=1}^{I} [(e^{\sum_j \alpha_j A_{jit} + \sum_k \beta_k B_{kt} F_{it}}) \cdot F_{it}^{1-\sigma}]}$

Therefore the "dynamic problem" can be shown as (3.36).

Minimize total cost =

$$\sum_{t=1}^{T}\sum_{i=1}^{I}\frac{1}{(1+\beta)^t}\left[\frac{12(e^{\sum_j \alpha_j A_{jit}+\sum_k \beta_k B_{kt} F_{it}})\cdot F_{it}^{1-\sigma}\cdot M_t \cdot VC_{it}}{L_{it}\cdot D_i \cdot \sum_{i=1}^{I}\left[(e^{\sum_j \alpha_j A_{jit}+\sum_k \beta_k B_{kt} F_{it}})\cdot F_{it}^{1-\sigma}\right]}\right] + FC_{1it} + FC_{2it}\cdot {x_{it}}^2 + FC_{3it}\cdot x_{i,t-1}^2 - (FC_{2it} + FC_{3it})\cdot x_{i,t}\cdot x_{i,t-1} \qquad (3.36)$$

where β = discount rate

Choice variables: $X_1, X_2, X_3, \ldots, X_T$

Subject to

1) Cash supply = Cash demand (in equilibrium)
2) The denomination structure follows binary-decimal triplets (1-2-5-10-20-50)
3) Given boundary from D-Metric
    (1) Lowest coin denomination
    (2) Transition between coins and banknotes which is identified by lowest banknote denomination
    (3) Highest banknote denomination
4) Cash payment efficiency criteria
5) Given full series denomination structure (D)
6) Other assumptions (for numerical analysis)

CHAPTER 4

CASH PAYMENT EFFICIENCY

Before we go to the empirical study and numerical analysis in chapter 5, cash payment efficiency will be re-explained in detail. It should be realized that not only cost is of concern, but also the convenience of cash use which would be proxied by cash payment efficiency is another intention to determine optimal denomination structure. In this chapter, cash payment efficiency is discussed in order to be applied to a numerical model specification.

As discussed in section 3.5 of chapter 3, cash payment efficiency can be measured in two ways, according to the average number of minimum tokens and the average number of efficient schemes. In order to apply them to a numerical analysis, the following assumptions are assigned in this research for simplicity:

1) Payers and payees have all denominations in their wallets in an unlimited amount. This is the simple and original version of Cramer's model (Cramer, 1983: 299-303).

2) People are assumed to make efficient payments according to the principle of least effort. In other words, transactors are basically expected to satisfy the transaction with a minimum number of monetary items.

3) The cash payment profile is assumed for simplicity to be uniformly distributed with equal chances for all payment amounts, and the weighted average is therefore not required for a calculation.

With all of the above assumptions, the numerical examples and interesting points about cash payment efficiency can be shown and discussed as follows:

4.1 Examples According to the Principle of Least Effort

4.1.1 With 1-2-4-8-16-32-64 Series

1) To pay 11 baht:

Number of minimum tokens = 3

Number of efficient payment schemes = 3

(1) 8+2+1

(2) 8+4-1

(3) 16-4-1

2) To pay 13 baht:

Number of minimum tokens = 3

Number of efficient payment schemes = 3

(1) 8+4+1

(2) 16-2-1

(3) 16+1-4

3) To pay 19 baht:

Number of minimum tokens = 3

Number of efficient payment schemes = 2

(1) 16+2+1

(2) 16+4-1

4) To pay 45 baht:

Number of minimum tokens = 4

Number of efficient payment schemes = 5

(1) 32+8+4+1

(2) 32+16-2-1

(3) 32+16+1-4

(4) 64-16-2-1

(5) 64+1-16-4

5) To pay 51 baht:

Number of minimum tokens = 4

Number of efficient payment schemes = 5

(1) 32+16+2+1

(2) 32+16+4-1

(3) 64-8-4-1

(4) 64+2+1-16

(5) 64+4-16-1

4.1.2 With 1-2-5-10-20-50-100 Series

1) To pay 35 baht:

Number of minimum tokens = 3

Number of efficient payment schemes = 4

(1) 20+10+5

(2) 20+20-5

(3) 50-10-5

(4) 50+5-20

2) To pay 37 baht:

Number of minimum tokens = 4

Number of efficient payment schemes = 6

(1) 20+10+5+2

(2) 20+20-2-1

(3) 20+20+2-5

(4) 50-10-2-1

(5) 50+5+2-20

(6) 50+2-10-5

4.1.3 With 1-2-5-20-50-100 Series (without 10)

1) To pay 28 baht:

Number of minimum tokens = 3

Number of efficient payment schemes = 1

i.e. 50-20-2

2) To pay 31 baht:

Number of minimum tokens = 3

Number of efficient payment schemes = 1

i.e. 50+1-20

4.2 The Number of Denominations, the Spacing Factor, and Cash Payment Efficiency

The structure of various currency types or large number of denominations is likely to have low average space (if the boundary is restricted), which is expected to provide high cash payment efficiency in two aspects. With low average space, the number of tokens used for payment tends to be small because the transactors are able to make efficient payments with minimum tokens from a dense denomination structure. On the other hand, the number of efficient payment schemes tends to be larger because there are a lot of alternatives that can increase the probability of the payments being formed with minimum tokens. In sum, the structure with high cash payment efficiency is expected to have many denominations with a small average spacing factor.

In order to determine the relationship between the number of denominations, the spacing factor, and cash payment efficiency, we begin with a simple example which shows all combinations of the denomination structures, with a start and end value at 1 and 10 respectively. The full series has four denominations, which are 1, 2, 5, and 10. By varying the series inside 1-10 boundaries, we obtain four possible scenarios of denomination structures, which are structures no.1 – 4 in Table 4.1. By applying the algorithm particularly constructed for the cash payment efficiency analysis in this research by simulating cash payment amounts from 1 to 50 with unit steps, the cash payment efficiency can be obtained. Assuming a uniform distribution of cash payment amount, the mean averages of the number of minimum tokens and number of efficient schemes are able to represent the two aspects of efficiency, as shown in Table 4.1.

Table 4.1 Denomination Structures in the Range from 1 to 10

No.	Structure	No. of denominations	Spacing factor	No. of minimum tokens			No. of efficient payment schemes		
				min	max	average	min	max	average
1	1-2-5-10	4	2.2	1	6	3.6	1	2	1.2
2	1-5-10	3	3.5	1	7	4.0	1	2	1.1
3	1-2-10	3	3.5	1	7	4.0	1	2	1.1
4	1-10	2	10.0	1	9	5.0	1	1	1.0

Note: The efficiency value is calculated from cash payment amount 1, 2, 3, …, 50.

From Table 4.1, it can be seen that the structure with a high number of denominations and a low spacing factor has high cash payment efficiency in both aspects without any contradiction. In fact, if the boundaries of the denomination structure (the lowest and highest denominations) are controlled, the number of denominations and spacing factor will be negatively correlated with each other. Moreover, the average number of minimum tokens and average number of payment schemes seem to be also negatively correlated with each other, which show the same direction of cash payment efficiency between the two aspects (the high cash payment efficiency comes from low average number of minimum tokens and a high number of efficient payment schemes). It can be therefore found at the beginning that cash payment efficiency (in both aspects) is a function of either the number of denominations or the spacing factor for the structures with a controlled boundary. However, there might be some cases where the structure with a higher number of denominations and a lower average spacing factor uses a smaller number of minimum tokens but also yields a smaller number of efficient payment schemes. Two examples are raised to shed light on this,

1) The first example is a comparison between structures 1-2-5-10 V.S.1-2-10 for a 6-baht payment.

Structure #1 (1-2-5-10):

Number of minimum tokens = 2

Number of efficient payment schemes = 1 i.e. 5+1

Structure # 2 (1-2-10):

Number of minimum tokens = 3

Number of efficient payment schemes = 2 i.e. 2+2+2 and 10-2-2

Which structure is more efficient for a 6-baht payment?

2) Another example is the comparison between the structures 1-2-5 and 1-2-10 for various ranges of payments.

Table 4.2 Cash Payment Efficiency Comparison between Structure 1-2-5 and 1-2-10

Range of payment amounts	**Average no. of minimum tokens**		**Average no. of efficient payment schemes**	
	1-2-5	**1-2-10**	**1-2-5**	**1-2-10**
1, 2, 3, …, 50	5.9	4.0	1.4	1.1
1, 2, 3, …, 100	10.9	6.5	1.4	1.1
1, 2, 3, …, 200	20.9	11.5	1.4	1.1
1, 2, 3, …, 500	50.9	26.5	1.4	1.1

At the beginning, there might be a question concerning how the range of payment amounts should be identified. We therefore simulate various scenarios of payment range and verify the two aspects of cash payment efficiency according to two denomination structures. The results show that the various payment ranges (four cases) simply simulated by adjusting the end value of the payment amount do affect the first aspect (minimum tokens) but not the second aspect (efficient payment schemes). However, the rank of efficiency of both aspects does not change. All scenarios indicate that 1-2-10 is more efficient than 1-2-5 for the first aspect (greater average number of minimum tokens) but less efficient for the second aspect (smaller average number of efficient schemes). The comparison result between the two structures shows the contradiction between the two aspects of cash payment efficiency. It is therefore difficult to conclude which structure is more efficient. It can be implied that it is not necessary for the two aspects of cash payment efficiency to be negatively correlated.

In the next sections, the relationship between the two aspects of cash payment efficiency will be investigated, followed by a discussion of the payment range or boundary to be used for cash payment efficiency calculation, together with its effects.

4.3 The Relationship between Two Aspects

According to Table 4.1, the two aspects of cash payment efficiency seem to be negatively correlated, i.e. a structure with a low average number of minimum tokens tends to yield a high average number of efficient payment schemes, which is a win-win case (the high cash payment efficiency is expressed by the small value of the average number of minimum tokens and the large value of the average number of efficient payment schemes). However, it can be seen from Table 4.2 that there are still some cases which are in non-compliance with the negative correlation and generate a trade-off between two aspects. In other words, we can see structures with a lower average number of minimum tokens but also with a lower average number of efficient schemes.

To corroborate the assertion, the pairs of two aspects of efficiency were collected from different denomination structures with various payment ranges or boundaries. A scatter diagram between the two aspects was prepared and is shown as Figure 4.1. We can see the thorough spread of data and negative linear correlation with a 15.7% fit to the data (R^2=0.157), which is a very low explanation. This imperfect relationship between the two aspects indicates some contradictory cases in the structures; for example, the structure with lower average number of minimum tokens also gives a lower number of efficient payment schemes and vice versa. It is therefore ambiguous to rank the denomination structures according to the cash payment efficiency perspective. In order to avoid this problem, the term "cash payment efficiency" is mainly emphasized regarding the average number of minimum tokens (first aspect) rather than the average number of efficient schemes (second aspect). First priority is given to the first aspect because the definition of efficient payment according to the principle of least effort is consistent with a payment with minimum tokens, which is in line with the efficiency in the first aspect. The second aspect is then realized to be subsequent criteria for efficient payment.

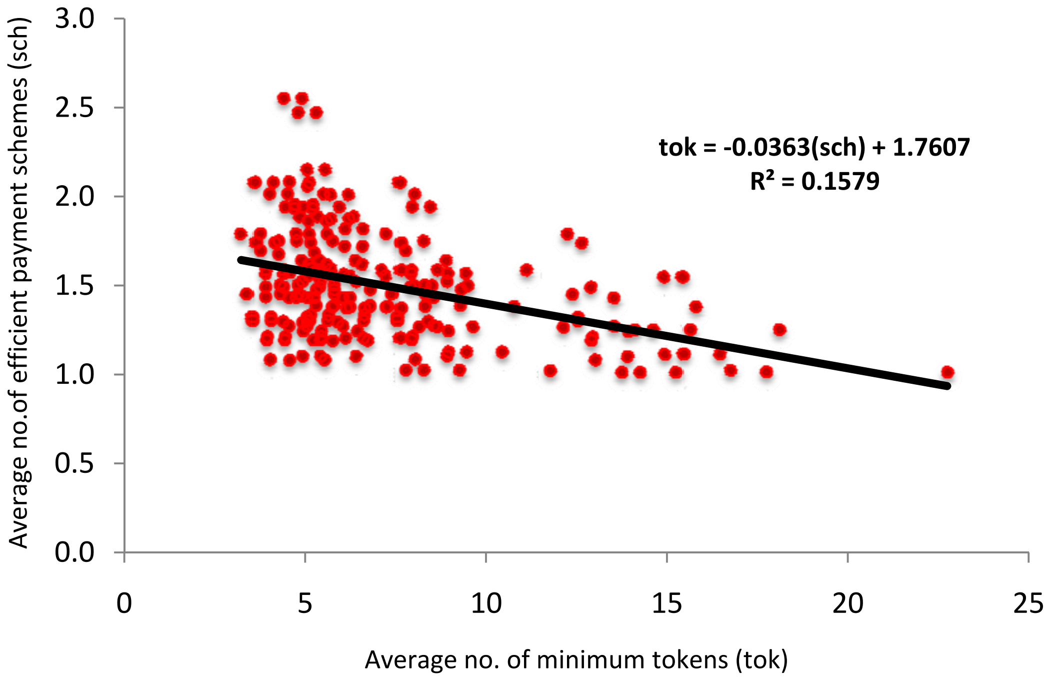


Figure 4.1 The Relationship between two Aspects of Cash Payment Efficiency

4.4 Boundary Effects

The boundary effect in this section refers to the impacts of the range of payment amount on the cash payment efficiency. Each payment range covers the start value, the end value, and the step between consecutive payments. We use the format "start-step-end" to represent the start value, the step between consecutive payments and the end value, respectively. For example, if we calculate the average efficiency from determining payment amount 5, 10, 15,….., 2000, the start-step-end is 5-5-2000.

The start value should be the minimum payment amount obtained from the cash payment profile. The step should be the least payment unit to be increased or decreased. Finally, the end value should be the maximum payment amount obtained from the cash payment profile. In this study, the cash payment profile is assumed to be uniformly distributed for simplicity. Moreover, the step value is assumed to be equal to the start value. Finally, the end value is assumed to be a multiple(s) of the highest denomination value.

We begin with the cash payment efficiency analysis of structure 1-2-5-10-20, with the variety of payment ranges in line with the assumption earlier discussed. The results are shown in Figures 4.2 and 4.3.

From Figure 4.2, it can be seen that the factors affecting the average number of minimum tokens (first aspect) represent the end value, which are positively related to each other. It is simple logic that the higher the payment amount is, the larger the number of minimum tokens that will be used, resulting in the average value being higher. However, the start and step values seem not to significantly affect the average number of minimum tokens. Figure 4.3 shows that the start and step values have an impact on the average number of efficient payment schemes (second aspect), while the end value seems not to affect it.

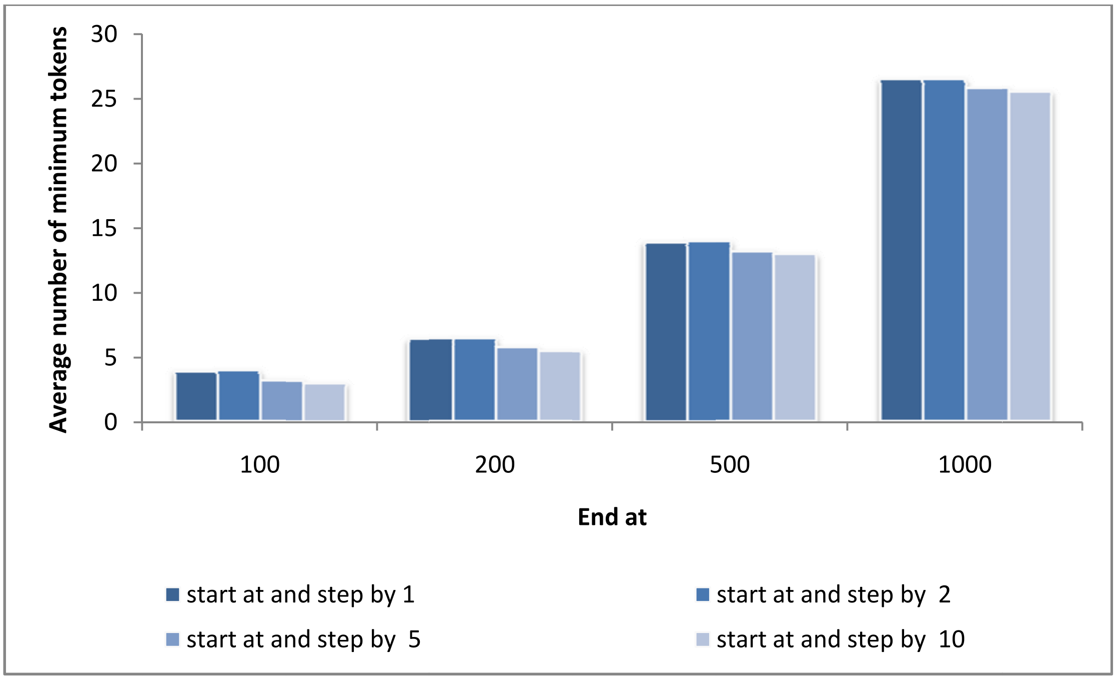


Figure 4.2 Cash Payment Efficiency (first aspect) of the Structure 1-2-5-10-20 Varied by Ranges of Payment Amounts

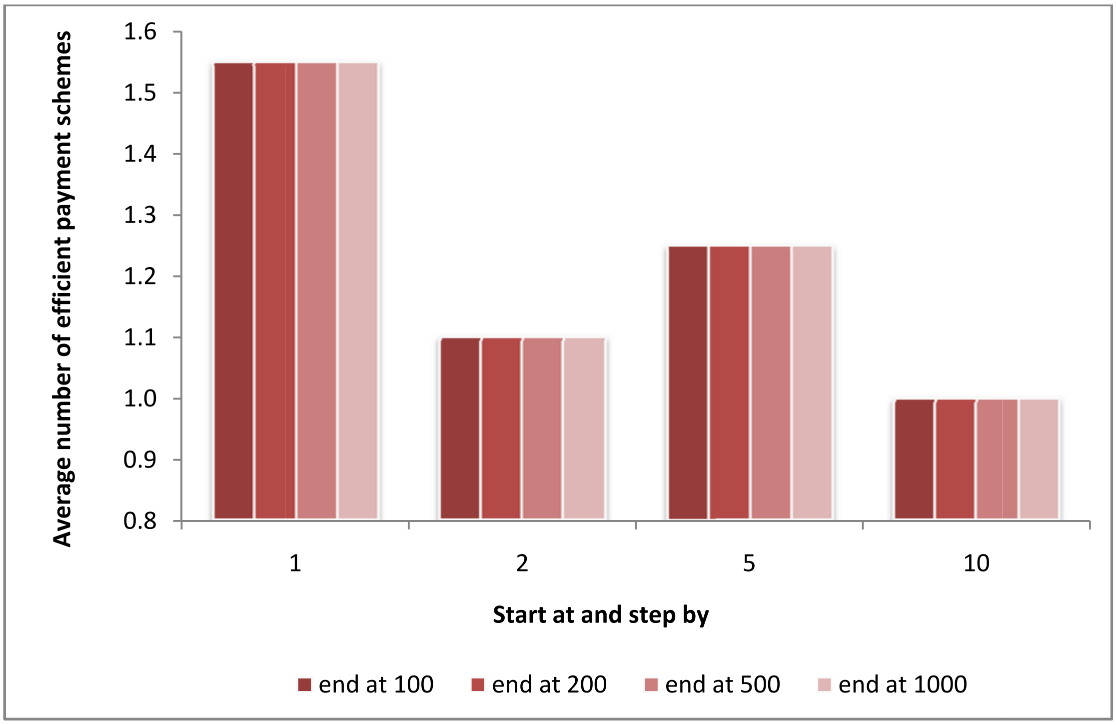


Figure 4.3 Cash Payment Efficiency (second aspect) of the Structure 1-2-5-10-20 Varied by Ranges of Payment Amounts

In order to make sure that the findings are valid without loss of generality, the denomination structure of full series is extended to contain 10 denominations, which are 1, 2, 5, 10, 20, 50, 100, 200, 500, and 1000. Three denomination structures, which are full series, series without 2 and series without 2 and 5, are examined with the end value of payment amount simulated until 3000. However, the start and step values are fixed at 1, which is the least unit of currency denomination. Figure 4.4 shows the results, which express the average minimum tokens as a function of the end value in the payment ranges. All of the structures in the three panels confirm the conclusion that the end value affects cash payment efficiency in the first aspect but not the second aspect. Comparing the three structures, the full series yields the highest payment efficiency in both aspects (lowest average number of minimum tokens and highest average number of efficient payment schemes). The second and third ranks are the structure without 2 and the structure without 2 and 5, respectively. These results are in line with the beginning hypothesis—that the structure with the highest number of denominations and/or lowest average spacing factor would probably be the most efficient for the two aspects without any contradictions. Looking inside each panel in Figure 4.4 for the individual structure, each aspect of cash payment efficiency is discussed as follows.

4.4.1 The First Aspect: Average Number of Minimum Tokens

Before we continue to discuss the topic in this section, there is one word to be clarified. “The loop of payment range″ is defined as the round of cash payment amounts with end values equal to the multiple(s) of the highest denomination value. For instance, suppose the structure is 1-2-5, the payment range 1-1-20 has 4 loops inside, which are 1,2,…,5 / 6,7,…,10 / 11,12,…, 15 / 16,17,…,20. For another example, suppose the structure is 1-5-10-50, the payment range 5-5-150 has 3 loops inside, which are 5,10,…,50 / 55,60,…,100 / 105,110,…, 150.

Figure 4.4 shows at most three loops (payment amount 3000 baht with 1000 baht per round). It can be found that the average minimum tokens increases by 0.5 when the payment range is extended by one loop. For example, in panel A, which is a full series, the payment range with one, two, and three loops shows the average number of minimum tokens to be 3.89, 4.39 and 4.89, respectively, which step up by

0.5 per one additional loop. The average number of minimum tokens then is expected to be equal to 5.89 if the end value is extended to 5000. This is also true for the structures in panel B and C. The reason is based on the assumption that the cash payment distribution is uniform. We can see that with every one loop increment, the minimum tokens used for each payment amount would be one token more (by adding the highest denomination). The average value therefore steps up by 0.5 per round. If the payment range is extended by three additional loops, the average number of minimum tokens will increase by (0.5)(3) = 1.5. Considering the structure 1-2-5 in Table 4.2, it can be seen that cash payments increase 5 baht per round, which is the highest denomination value. For payment range 1-1-50, which covers 9 loops appended from the first one (1-1-5), the average number of minimum tokens equals 5.9, implying, that the range 1-1-5 will yield average number of minimum tokens equal to 5.9 - (0.5)(9) = 1.4. For payment range 1-1-500, which covers 99 loops appended from the first one, the average number of minimum tokens is therefore equal to 1.4 + (0.5)(99) = 50.9, which is in line with the figure in Table 4.2.

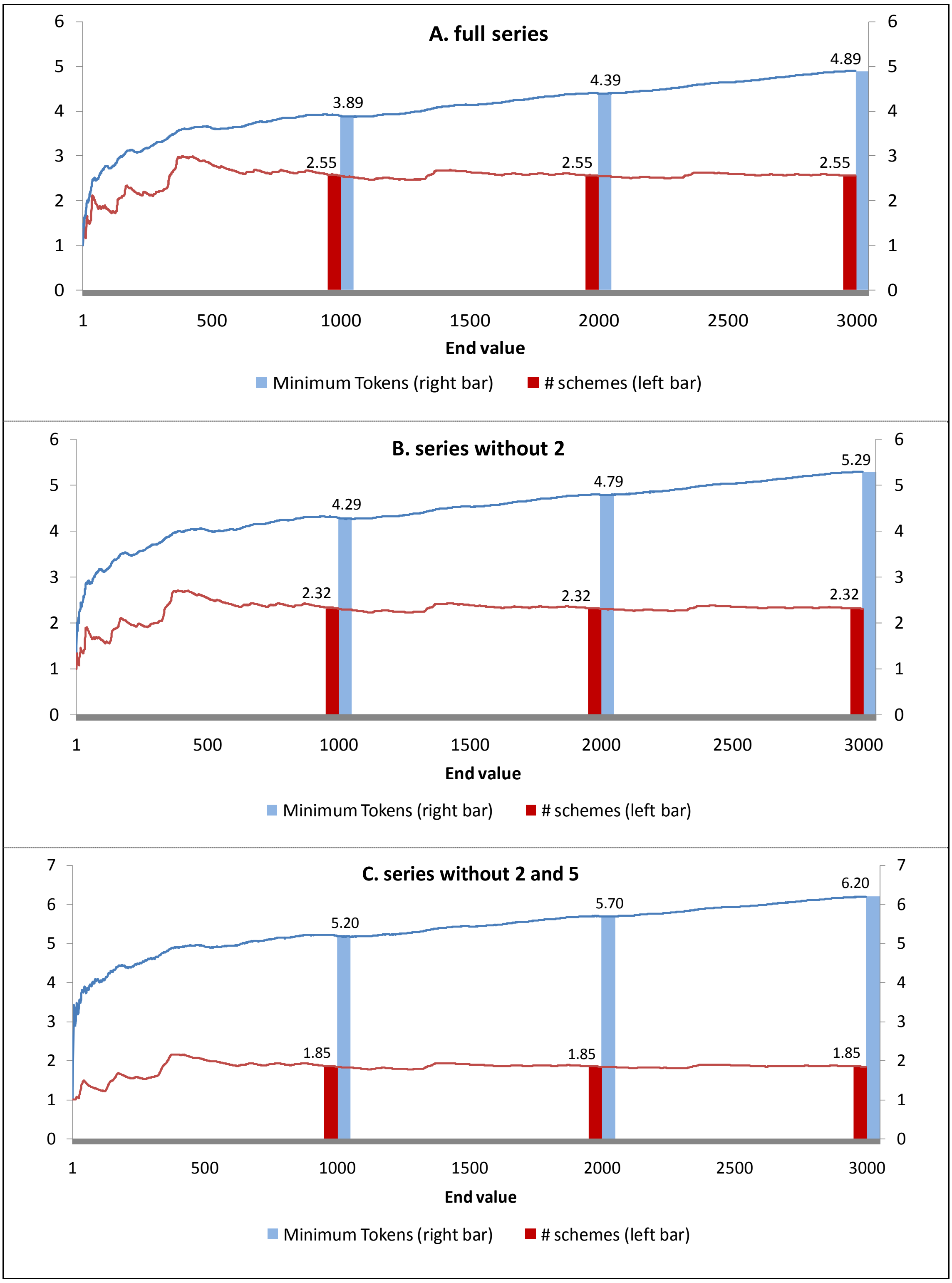


Figure 4.4 "End Value" Effect on Cash Payment Efficiency

Note: The full series denotes the structure 1-2-5-10-20-50-100-200-500-1000.

For the end value which lies inside a loop, e.g. end value = 24 in structure 1-2-5, the average number of minimum tokens tends to linearly increase with a bit of a fluctuation. This is shown in Figure 4.5, which is an example of payment amount of 5 baht per round (according to the highest denomination = 5 baht). It can be observed that the more loops the payment range contains, the more linearity of the average number of minimum token profile there will be. Moreover, it confirms our finding that the average number of minimum tokens would increase by 0.5 for every one loop increment of payment range, i.e. 1.8, 2.3, 2.8, …, 11.3.

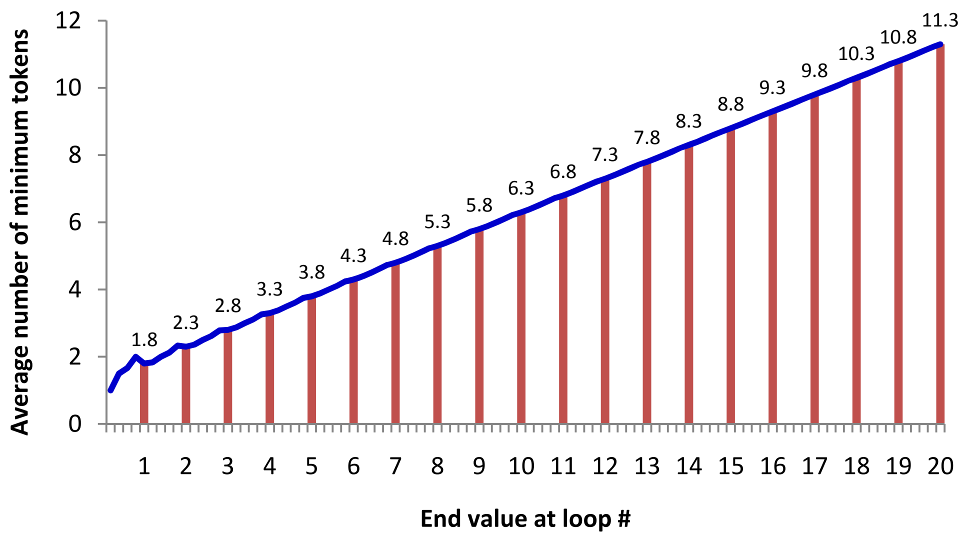


Figure 4.5 The Impact of End Value on 1st Aspect Efficiency

Note: Each loop is assumed to be composed of five payment amounts with unit steps e.g. 1,2,3,4,5.

4.4.2 The Second Aspect: Average Number of Efficient Payment Schemes

Considering Figure 4.4, it can be found that with every one loop increment of payment range, the average numbers of efficient payment schemes are always constant. For example, in panel A, which is a full series, the 1st, 2nd and 3rd rounds show the same value of average number of efficient payment schemes, which is equal to 2.55. This is also true for the structures in panel B and C. The reason is based on the assumption that the cash payment distribution is uniform. We can see that with each one loop increment of payment range, the number of efficient payment schemes repeats the same cycle because an additional token would be the highest denomination, which has only one way to add it. The average number of efficient payment schemes is therefore fixed for every one loop increment.

However, for the end value which lies inside a loop, the average number of efficient payment schemes fluctuates around the constant value and tends to converge to that value when there are more loops inside the payment range (see Figure 4.6 for an example of a payment amount of 5 baht per round).

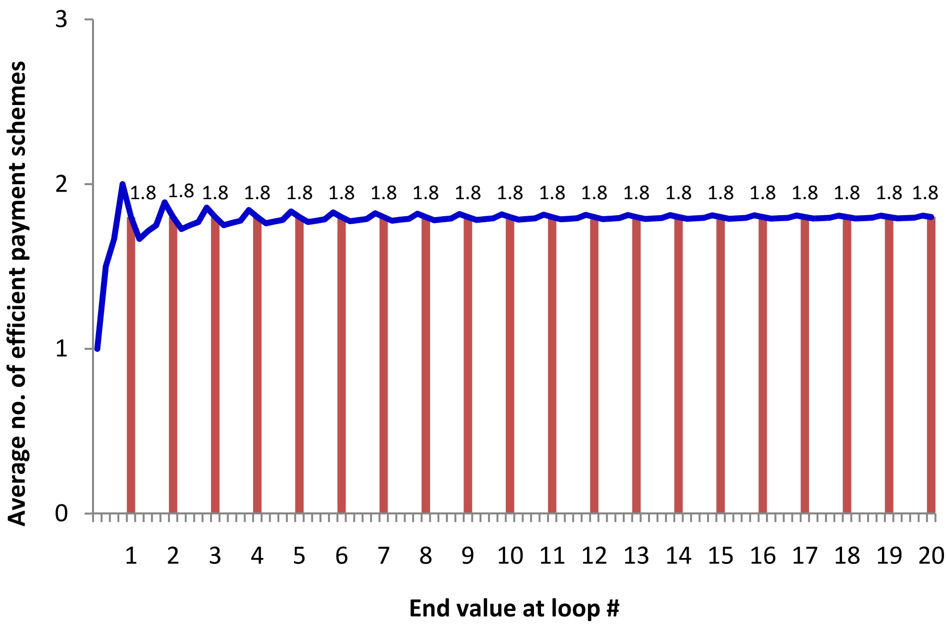


Figure 4.6 The Impact of End Value on 2^{nd} Aspect Efficiency

Note: Each loop is assumed to be composed of five payment amounts with unit steps.

With this finding concerning the boundary effect, together with the assumption of uniform distribution of cash payment and the control of the start and step to be equal, the end value should be higher than the value of the highest denomination to cover the role of that denomination. Moreover, the magnitude of the end value affects cash payment efficiency, especially for the first aspect, but it does not matter when being applied for comparison purposes. However, the best way to identify boundaries or the payment range is to study the cash payment profile, which allows the weighted average for a calculation of cash payment efficiency.

4.5 Wider Simulations

If the denomination structure of the full series is extended to contain 10 denominations, which are 1, 2, 5, 10, 20, 50, 100, 200, 500, and 1000, together with cash payment amounts from 1 to 3000 instead of 50, there will be some interesting points and in-depth findings, as shown in Table 4.3.

Table 4.3 Cash Payment Efficiency in Various Denomination Structures (Based on Full Series 1-2-5-10-20-50-100-200-500-1000) Calculated by 1-1-3000 Payment Range

	Structure	No. of Denominations	No. of Run(s)	Average Spacing Factor	Average No. of Minimum Tokens	Average No. of Efficient Payment Schemes
1.	full	10	1	2.17	4.89	2.55
2.	without2	9	2	2.50	5.29	2.32
3.	without 5	9	2	2.50	5.20	1.92
4.	without 20	9	2	2.50	5.19	1.83
5.	without50	9	2	2.50	5.19	1.93
6.	without200	9	2	2.50	5.20	1.96
7.	without 500	9	2	2.50	5.29	2.47
8.	without 2, 5	8	2	3.29	6.20	1.85
9.	without 2, 20	8	3	2.93	5.59	1.68
10.	without 2, 50	8	3	2.93	5.59	1.75
11.	without 2, 200	8	3	2.93	5.60	1.77
12.	without 2, 500	8	3	2.93	5.69	2.24
13.	without 5, 20	8	3	2.93	5.58	1.74
14.	without 5, 50	8	3	2.93	5.50	1.42
15.	without 5, 200	8	3	2.93	5.50	1.43
16.	without 5, 500	8	3	2.93	5.60	1.87
17.	without 20, 50	8	2	3.29	6.09	1.50
18.	without 20, 200	8	3	2.93	5.50	1.44
19.	without 20, 500	8	3	2.93	5.59	1.76
20.	without 50, 200	8	3	2.93	5.58	1.86
21.	without 50, 500	8	3	2.93	5.59	1.79
22.	without 200, 500	8	2	3.29	6.20	1.87
23.	without 2, 20, 200	7	4	3.50	5.90	1.32
24.	without 2, 20, 500	7	4	3.50	5.99	1.62
25.	without 2, 50, 200	7	4	3.50	5.98	1.68
26.	without 2, 50, 500	7	4	3.50	5.99	1.62
27.	without 5, 20, 200	7	4	3.50	5.88	1.31
28.	without 5, 20, 500	7	4	3.50	5.98	1.68
29.	without 5, 50, 200	7	4	3.50	5.88	1.31
30.	without 5, 50, 500	7	4	3.50	5.90	1.32

It can be observed from Table 4.3 that:

1) The structure with same number of denominations may have different average spacing factors, especially for the wide range of denomination structures. However, the relationship between the number of denominations and the average spacing factor is still negatively correlated if the highest and lowest denominations are controlled.

2) It can be confirmed that there are some cases in which two aspects of cash payment efficiency are contradicted, e.g. structure number 8 and 9, and structure number 18 and 19. The contradiction will yield an ambiguous solution as to which structure gives the higher cash payment efficiency unless there is a rule to give first priority to the first aspect.

3) The structure with various denominations and low average spacing seems to be efficient, as earlier discussed in section 4.2. This is because people have several choices for payment selection with minimum tokens. However, there might be a case where a structure with lower denominations and a larger average spacing factor has higher cash payment efficiency, for example, structure number 25 compared to number 17. Here we can observe that the structure without two consecutive denominations would have less payment efficiency compared to the other cases. This implies that there should be other structure characteristics beside the spacing factor and the number of denominations which involve the cash payment efficiency. According to the above observation, the number of "runs" is therefore introduced to separate the case of consecutive denominations, which are simultaneously skipped in the structure. The "run" is defined as a subgroup without any elimination from the full series. For example, suppose the full series is 1-2-5-10-20-50, it can be found that:

(1) Structure 1-10-20-50 has the number of runs equal to 2.

(2) Structure 1-2-10-50 has the number of runs equal to 3.

Comparing the structures with the same number of denominations, the structures with the higher number of runs would have the lower average spacing factor and would tend to yield higher cash payment efficiency, especially for the first aspect efficiency, which is the first priority of cash payment efficiency determination, e.g. structure number 8 compared to number 12.

Comparing the structures with different number of denominations, only average spacing factor is not sufficient to decide which structure would yield higher cash payment efficiency, e.g. structure number 17 compared to number 25, as mentioned earlier. Structure number 25 has a lower number of denominations and a higher average spacing factor but a higher number of runs; therefore it yields higher cash payment efficiency rather than structure number 17.

4.5.1 The Regression Analysis

In order to examine the factors affecting cash payment efficiency, the basic multi-regression model is introduced. The purpose of regression is not to predict cash payment efficiency from various structure scenarios because we would already know the exact values that can be obtained from the algorithm. However, the basic multi-regression is applied to determine simple patterns of cash payment efficiency functions and to confirm the hypotheses on the relationship directions of relevant factors regarding cash payment efficiency.

It should be remembered that for any given denomination structure, the cash payment efficiency can be obtained in two ways: with the average number of minimum tokens (first aspect) and the average number of efficient payment schemes (second aspect). However, each structure will yield different efficiency values if they come from different sets of payment amounts used for calculation. It can be concluded that the variation of cash payment efficiency comes from two main groups: its structure and the set of payment amounts used for calculation.

1) The denomination structure

The characteristics of a structure which affect cash payment efficiency are the lowest value of the currency, the highest value of the currency, the average spacing factor, the number of denominations regardless how many coins or banknote denominations, and the number of runs. In this research, the boundary of the denomination structure is given; therefore the spacing factor, the number of denominations, and the number of runs (which actually may have some correlations with each other) are the factors affecting the efficiency value. They were taken into the models with the hypothesis that a low spacing factor, many denominations, and a large number of runs would yield high payment efficiency because there would be various alternatives in terms of making a payment by minimum tokens with several schemes.

2) The set of payment amounts used for calculation that is ‘start-step-end’ as earlier defined in section 4.4. As we assume that the step is set to be equal to the start value, the start value and end value are two factors affecting the efficiency value and were taken into the models with the hypothesis from our observation—that the start value has a negative relationship with both aspects of cash payment efficiency, whereas the end value has a positive relationship to the first aspect of cash payment efficiency but do not significantly affect the second aspect of cash payment efficiency.

According to the above background, two models were set up to help explain the model for the first and second aspects of cash payment efficiency. Each model will regress cash payment efficiency on five factors, which are the spacing factor, the number of denominations, the number of runs, start and end value. The mathematical models can be written as follows:

tokens = f(start, end, nodeno, run, spacing)
schemes = f(start, end, nodeno, run, spacing)

where

tokens is the average number of minimum tokens
schemes is the average number of efficient payment schemes
start is the start value of payment range used for efficiency calculation
end is the end value of payment range used for efficiency calculation
nodeno is the number of denominations
run is the number of runs
spacing is the average spacing factor

However, there are some limitations and scope to the regression as follows:

1) The full series contains 10 denominations, i.e. 1-2-5-10-20-50-100-200-500-1000

2) The cash payment profile is assumed to be uniformly distributed.

The functional forms of both models are assumed to be linear. Consequently, the two econometric models can be written as follows:

$$tokens_i = \alpha_0 + \alpha_1 \cdot start_i + \alpha_2 \cdot end_i + \alpha_3 \cdot nodeno_i + \alpha_4 \cdot run_i + \alpha_5 \cdot spacing_i + \varepsilon_i$$

$$schemes_j = \beta_0 + \beta_1 \cdot start_j + \beta_2 \cdot end_j + \beta_3 \cdot nodeno_j + \beta_4 \cdot run_j + \beta_5 \cdot spacing_j + \varepsilon_j$$

where ε_i and ε_j are the error terms

The results can be shown and interpreted as follows:

1) Minimum Tokens Model

$$\widehat{TOKENS} = 2.4812 - 0.1362START + 0.0005END +$$
$$(5.618^*) \quad (-37.023^*) \quad (160.973^*)$$

$$1.0889SPACING - 0.2300RUN - 0.1047NODENO$$
$$(75.421^*) \quad (-6.064^*) \quad (-2.757^*)$$

Adjusted $R^2 = 0.9956$

The figures in parentheses show the standard errors of estimators.

(*) denotes statistical significance at the 5% significance level.

This is a model for the cash payment efficiency in the first aspect, which is the average number of minimum tokens. It can be found that all factors can individually explain the average number of minimum tokens at the 5% significance level. The variation of the average number of minimum tokens can be explained by a model by 99.6%. All signs are in line with our hypotheses and observations from the previous figures.

The start value can explain the average number of minimum tokens in the opposite direction at the 5% significance level. However, in order to identify the possible structures, the start value has to be pre-determined from the least unit of payment via the cash payment profile.

The end value can explain the average number of minimum tokens in the same direction at the 5% significance level. This is consistent with the conclusion concerning the boundary effect discussed in section 4.4.

The average spacing factor can explain the average number of minimum tokens in the same direction at the 5% significance level. The higher spacing factor would increase the average number of minimum tokens, which decreases cash payment efficiency in the first aspect. This is consistent with the finding in section 4.2 and the observation in Table 4.3.

The number of runs can explain the average number of minimum tokens in the opposite direction at the 5% significance level. A greater number of runs would decrease the average number of minimum tokens, which increases cash payment efficiency in the first aspect. This is consistent with the observation in Table 4.3.

The number of denominations can explain the average number of minimum tokens in the opposite direction at the 5% significance level. The greater number of denominations would decrease the average number of minimum tokens, which favors cash payment efficiency in the first aspect. This is consistent with the finding in section 4.2 and the observation in Table 4.3.

2) Efficient Payment Schemes Model

$$\widehat{SCHEMES} = 0.3544 - 0.0425START - 0.0000END + $$
$$(0.943) \quad (-10.652^*) \quad (-0.172)$$

$$0.0029SPACING - 0.02087RUN + 0.1779NODENO$$
$$(0.260) \quad (-0.611) \quad (5.312^*)$$

Adjusted $R^2 = 0.5432$

The figures in parentheses show the standard errors of estimators.

(*) denotes statistical significance at the 5% significance level.

This is a model for the second aspect, which is the average number of efficient payment schemes. It can be found that only two factors can individually explain the average number of efficient payment schemes of at the 5% significance level. The variation of the average number of minimum tokens can be explained by a model only at 54.3%, which is lower than with the first model. All signs are in line with our hypotheses and observations from the previous figures.

The start value can explain the average number of efficient payment schemes in the opposite direction at the 5% significance level. Similar to the first model, to identify the possible structures, the start value has to be pre-determined from the least unit of payment via the cash payment profile. Therefore, this factor plays a minor role for the optimization problem in this study.

The end value cannot explain the average number of efficient payment schemes at the 5% significance level. This is consistent with the conclusion about the boundary effect discussed in section 4.4 and the observation in Figure 4.3.

The average spacing factor and the number of runs cannot explain the average number of efficient payment schemes at the 5% significance level. This can be supported by considering the numerical example in Table 4.3, in which the relationships between the average spacing factor, the number of runs, and the average number of efficient payment schemes seem to be ambiguous.

The number of denominations can explain the average number of minimum tokens in the same direction at the 5% significance level. The greater number of denominations would increase the average number of efficient payment schemes, which favors efficiency in the second aspect. This is consistent with the finding in section 4.2 and with the observation in Table 4.3.

According to the results from the two estimated models, the first model, which regresses the average number of minimum tokens on the relevant factors, has better performance for explicating cash payment efficiency rather than the second model. It can well explain the variation in the average number of minimum tokens by a straightforward linear function (with R^2 = 99.6%). The poorer fit of the second model probably comes from the inappropriate functional form which is assumed to be linear in variables. Moreover, there might be some unknown factors which can help explain the average number of minimum tokens but were not taken into the model. Fortunately, the average number of minimum tokens, which was determined in the first model, is the first priority emphasized for cash payment efficiency according to the principle of least effort. This confirms the simple logic that the denomination structure which has a large number of denominations with low average space and number of runs tends to be efficient according to the principle of least effort.

4.5.2 Symmetric Structures

Re-considering Table 4.3, it can be observed that some different structures have the same number of denominations, number of runs, and spacing factors. These structures can be categorized in five groups, as follows:

Group #1 structures number 2 to 7, which have a number of denominations, number of runs, and average spacing factor equal to 9, 2, and 2.50

Group #2 structures number 9 to16 and 18 to 21, which have a number of denominations, number of runs, and average spacing factor equal to 8, 3, and 2.93

Group #3 structures number 23 to 30, which have a number of denominations, number of runs, and average spacing factor equal to 7, 4, and 3.50

Group #4 structures number 8, 17 and 22, which have a number of denominations, number of runs and average spacing factor equal to 8, 2 and 3.29

Group #5 structure number 1 which is a full series with the number of denominations, number of runs, and average spacing factor equal to 10, 1, and 2.17

It can be seen that among the structures in the same group, there are some couples which give the same average numbers of minimum tokens, which is the cash payment efficiency in the first aspect. For example,

1) In group #1, structures 2 to 7 have 3 couples, i.e. structure 2 vs. 7, structure 3 vs. 6, and structure 4 vs. 5.

2) In group #3, structures 23 to 30 have 4 couples, i.e. structure 23 vs. 30, structure 24 vs. 26, structure 25 vs. 28 and structure 27 vs. 29.

The story behind this phenomenon is the symmetry of the structure. The term “symmetry” denotes the symmetric structure from left to right. For example, the full series 1-2-5-10: 1-2-10, and 1-5-10 are symmetrical with each other, the full series 1-2-5-10-20-50-100: 1-5-10-50-100, and 1-2-10-20-100 are symmetrical with each other, while 1-2-10-50-100 has no symmetrical structure because it is symmetric itself.

Note that the denomination structures which are in the same group have the same number of denominations, number of runs, and average spacing factor. According to the estimated models obtained in the last section, these three factors can well and nearly perfectly explain the average numbers of minimum tokens. Therefore, the structures in the same group would give the similar values of the first aspect of cash payment efficiency. However, these three factors partially explain the average number of efficient payment schemes, the structures in the same group are therefore not necessary to give similar values for the second aspect of cash payment efficiency.

In order to see how the structures those are symmetric with each other yield nearly equivalent cash payment efficiency, the number of minimum tokens and the number of efficient payment schemes for each payment, including average cash payment efficiency in both aspects, are expressed in Figure 4.7 and 4.8. Figure 4.7 shows the case of a simple structure with four denominations in full series, which is 1-2-5-10, and compares two structures that are symmetric with each other, i.e. 1-2-10 and 1-5-10. These two structures have the same number of denominations, number of runs, and average spacing factor equal to 3, 2 and 3.50, respectively. They are expected to give the same values of the average number of minimum tokens and similar value of the average number of efficient payment schemes (the algorithm shows the value of 2.5 and 1.1, respectively, with a 1-1-20 payment range).

As shown in Figures 4.7 and 4.8, it can be seen that the higher the end value is, the average cash payment efficiency converges to the same value between two structures in both aspects. The profiles of the minimum tokens and number of payment schemes are dominated by the full series structure (always lies below and above the profile of other structures for the profiles of minimum tokens and number of payment schemes, respectively). On the other hand, the profiles of 1-2-10 and 1-5-10, which are symmetric with each other, move up and down alternately to each other along payment amounts in equal amplitudes.

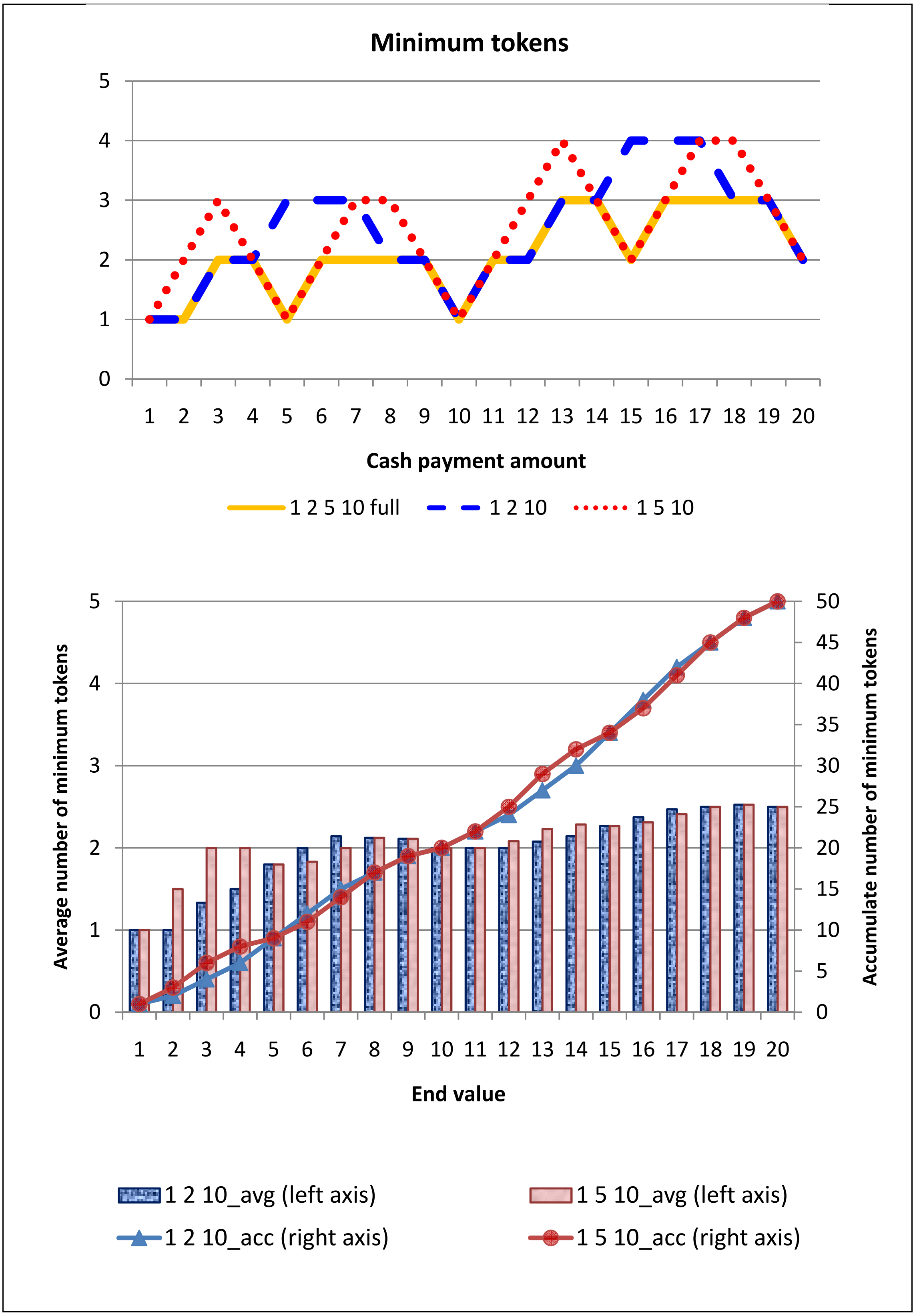


Figure 4.7 The Comparison of 1st Aspect Efficiency between Symmetric Structures

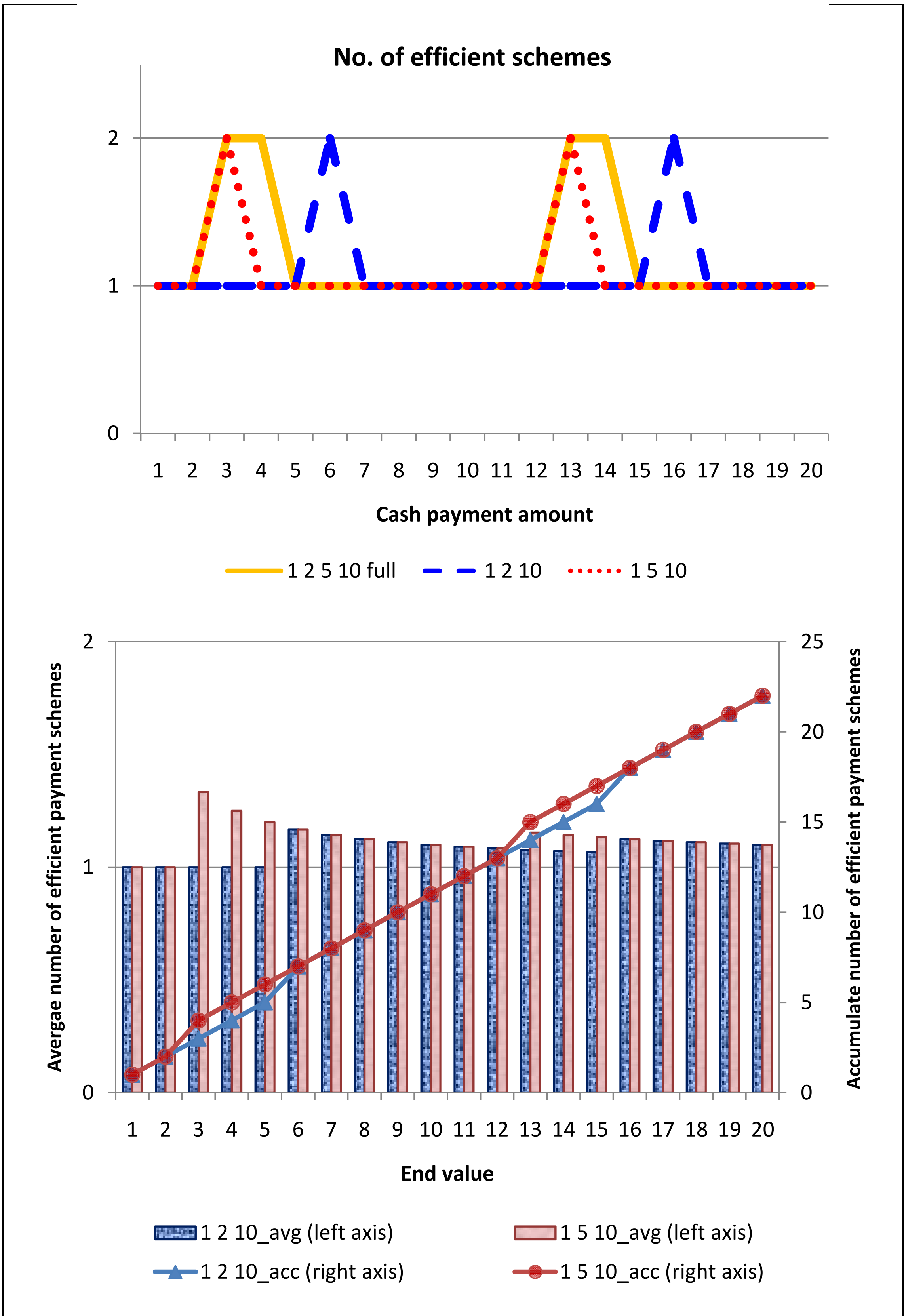


Figure 4.8 The Comparison of 2nd Aspect Efficiency between Symmetric Structures

Returning to full series 1-2-5-10-20-50-100, as shown in Table 4.3, it can be considered that the structure without 5 yields a cash payment efficiency nearly equal to the structure without 200, but the latter is slightly superior because of higher average number of efficient payment schemes, which is the second aspect of efficiency. It can be implied that, with cash payment efficiency concern regardless of cost and people's preferences, the two structures (without 5 and without 200) are nearly equivalent. This is not sensible because from a social view regarding both supply and demand, the two structures are different and cannot be absolutely substituted.

This is one reason why the analysis of cost, pricing pattern, payment pattern, consumers' preferences and behaviors are additionally required (besides the highest cash payment efficiency perspective) to find the solution for the optimal currency denomination. For example, if the payment amounts are usually in the range of 80-100 and always end with digit '5,' denomination 5 would be preferred to denomination 200. Consequently the structure without 200 is superior to the structure without 5.

4.6 Why 1-2-5?

According to the principle of least effort and the literature reviews in chapter 2, there are two main ideas regarding the optimal denomination structures in theory, which are the spacing of 2 (1-2-4-8-…) and the spacing of 3 (1-3-9-27-…). However, in practical use, there are two existing structures applied in the real world, which are binary-decimal triplets (1-2-5-10-…) and fractional-decimal triplets (1-2.5-5-10-…). This section will examine cash payment efficiency by assuming an infinite number of denominations with boundary of payment amounts (start-step-end) of 1-1-3000. Moreover, 1-4-7-10-… and 1-5-10-50-… are additional structures included in the structure comparison. The results are shown in Table 4.4 and Figure 4.9

Figure 4.9 shows two aspects of cash payment efficiency for the six structures. The circle markers denote the dominating structures (up to the left direction for a high average number of efficient payment schemes and a low number of minimum tokens) according to two aspects of cash payment efficiency. It can be seen that three structures, which are 1-2-4-8-…, 1-2-5-10,… and 1-3-9-27-…, would be candidates for an efficient denomination structure. In detail, structure 1-2-4-8-… is the structure with the lowest number of minimum tokens, which yields the highest cash payment efficiency according to the first aspect. 1-3-9-27-… is the structure with the highest number of efficient payment schemes which yields the highest cash payment efficiency according to the second aspect. 1-2-5-10-… is the structure that compromises between two aspects. Figure 4.10 shows that the ranking among the three dominating structures is unchanged regardless what the end value of the payment range is. However, if the average number of minimum tokens is the first priority to be determined for cash payment efficiency, as earlier assigned, 1-2-4-… and 1-2-5-10-… will be the top two structures which yield a similar value of cash payment efficiency (1-2-4-8-… is a bit better). Nevertheless, the reasonable arguments which motivate structure 1-2-5-10-… to be superior to 1-2-4-8-… may be:

1) the average number of payment schemes from structure 1-2-5-10-… is higher than that of structure 1-2-4-8-…

2) structure 1-2-5-10-… is compatible with the decimal currency system, which is applied for practical use in all countries in the world

3) cash in the 1-2-5-10-… series would be more convenient to use and to be counted and calculated by users because all denominations are factors of base-ten numbers, e.g. 10, 100, 1000, ….

4) in case that the cash payment profile shows a biased distribution toward the payment amount ending with '5' or '0', the structure 1-2-5-10-… is simply claimed to be more appropriate than 1-2-4-8-…

Table 4.4 The Properties of Various Structures and Their Cash Payment Efficiencies Calculated by 1-1-3000 Payment Range

Structure	No. of Denominations	No. of Run(s)	Average Spacing Factor	Average No. of Minimum Tokens	Average No. of Efficient Payment Schemes
1-2-4-8	infinite	1	2.00	4.37	2.38
1-3-9-27	infinite	1	3.00	5.25	4.66
1-2-5-10	infinite	1	2.17	4.46	2.96
1-2.5-5-10	infinite	1	2.17	4.87	2.90
1-5-10-50	infinite	1	3.50	5.90	1.32
1-4-7-10	infinite	1	2.39	4.74	1.13

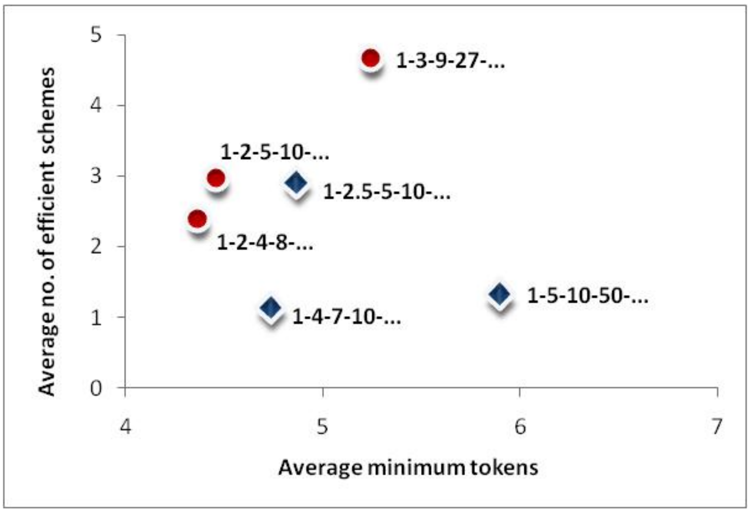


Figure 4.9 The Cash Payment Efficiency (in Two Aspects) of Various Structures Calculated by 1-1-3000 Payment Range

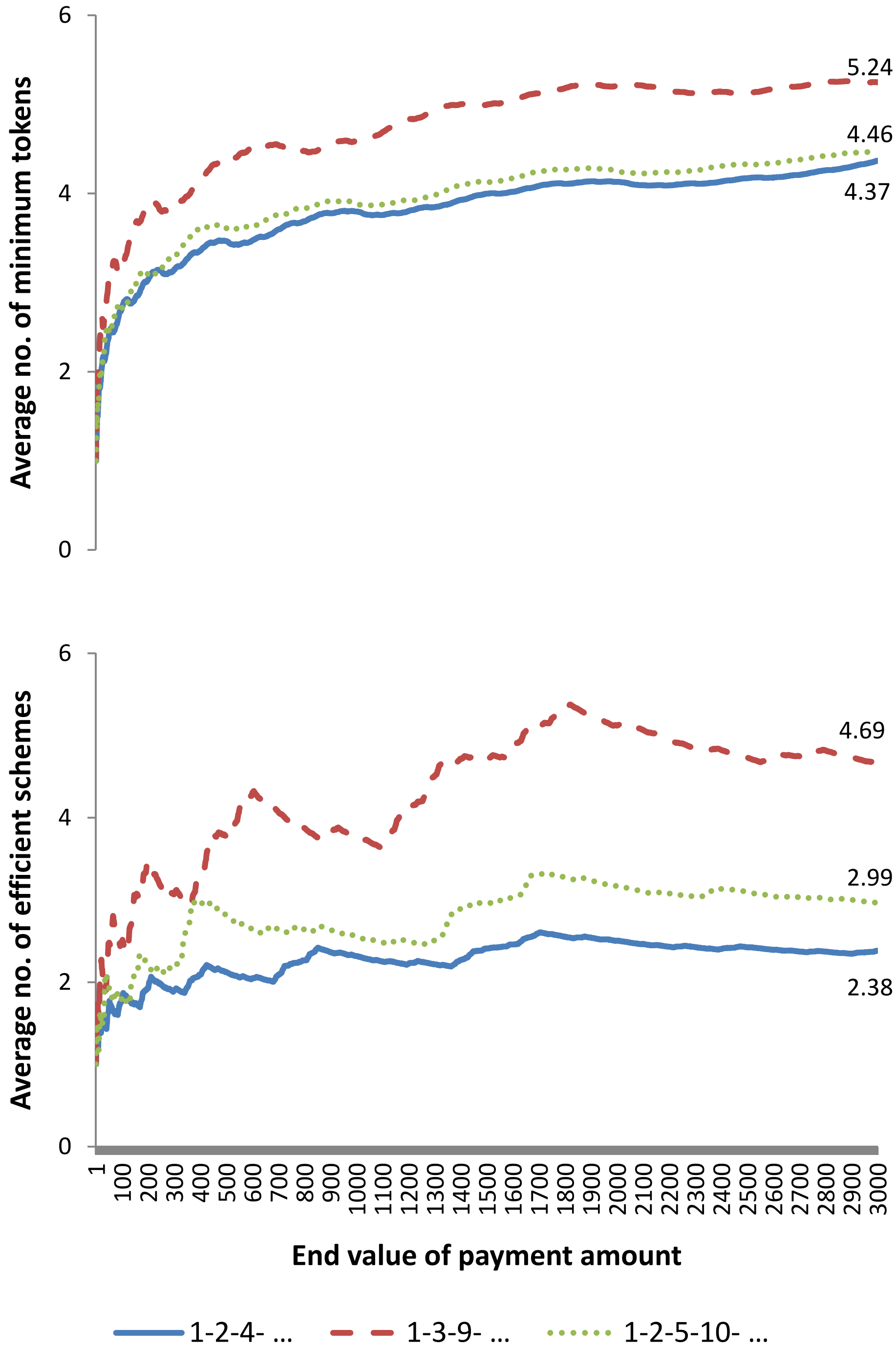


Figure 4.10 The Comparison of Cash Payment Efficiency between Three Structures Varied by End Value and Calculated by 1-1-3000 Payment Range

CHAPTER 5

EMPIRICAL STUDY AND NUMERICAL ANALYSIS

5.1 Overview

In this chapter, we numerically examine the banknote denomination structure of Thailand with two purposes. First, we will examine the currency denomination structure currently used in Thailand concerning whether it is optimal according to cost and cash payment efficiency perspectives. Static analysis is applied to serve this purpose and the years 2011-2015 constitute the period examined. Second, we applied dynamic analysis to answer the research question, "What should be the optimal currency denomination structure in Thailand?" In point of fact, changing the denomination structure is difficult to be applied and takes a long time to be completely carried out. This is why the optimal structure should be dynamically analyzed. The differences between static and dynamic are the following: 1. the linkage between periods such as costs for elimination and introduction of banknote denominations; 2. a discount rate taken into the model; and 3. the number of study periods, which leads to a bit of a difference in a model in terms of the objective function to be optimized. The empirical model specification is shown in section 5.4.1.

According to the conceptual framework and the theoretical model in chapter 3, the empirical analysis methodology is reorganized and clearly explained, as shown in Figure 5.1, which shows the components of an empirical framework. The methodology is based on the simulation-optimization technique[21]. The procedure can be divided into three main components expressed in the boxes with dash line borders. The first component is the estimation of banknote demand share by denomination. As described in chapter 3, the characteristic model is applied to estimate the demand

[21] Simulation-Optimization is defined as the process of finding the best input variable values from among all possibilities without explicitly evaluating each possibility.

function. The second component is the projection of exogenous variables and relevant factors to predict the banknote production plan by denomination, given a specific denomination structure. The last component is the cost minimization problem under a cash payment efficiency constraint. The currency denomination structures will be simulated under the given boundary and some assumptions, which are divided into four groups (Assumptions #1-4). Together with the two components obtained earlier, the possible candidates are selected by comparing total cost and cash payment efficiency. The structures which yield a minimum total cost with acceptable (high) cash payment efficiency are then the solutions[22]. The details for each component are illustrated in the following sections.

[22] There might be more than one solution according to the levels of acceptable (high) cash payment efficiency.

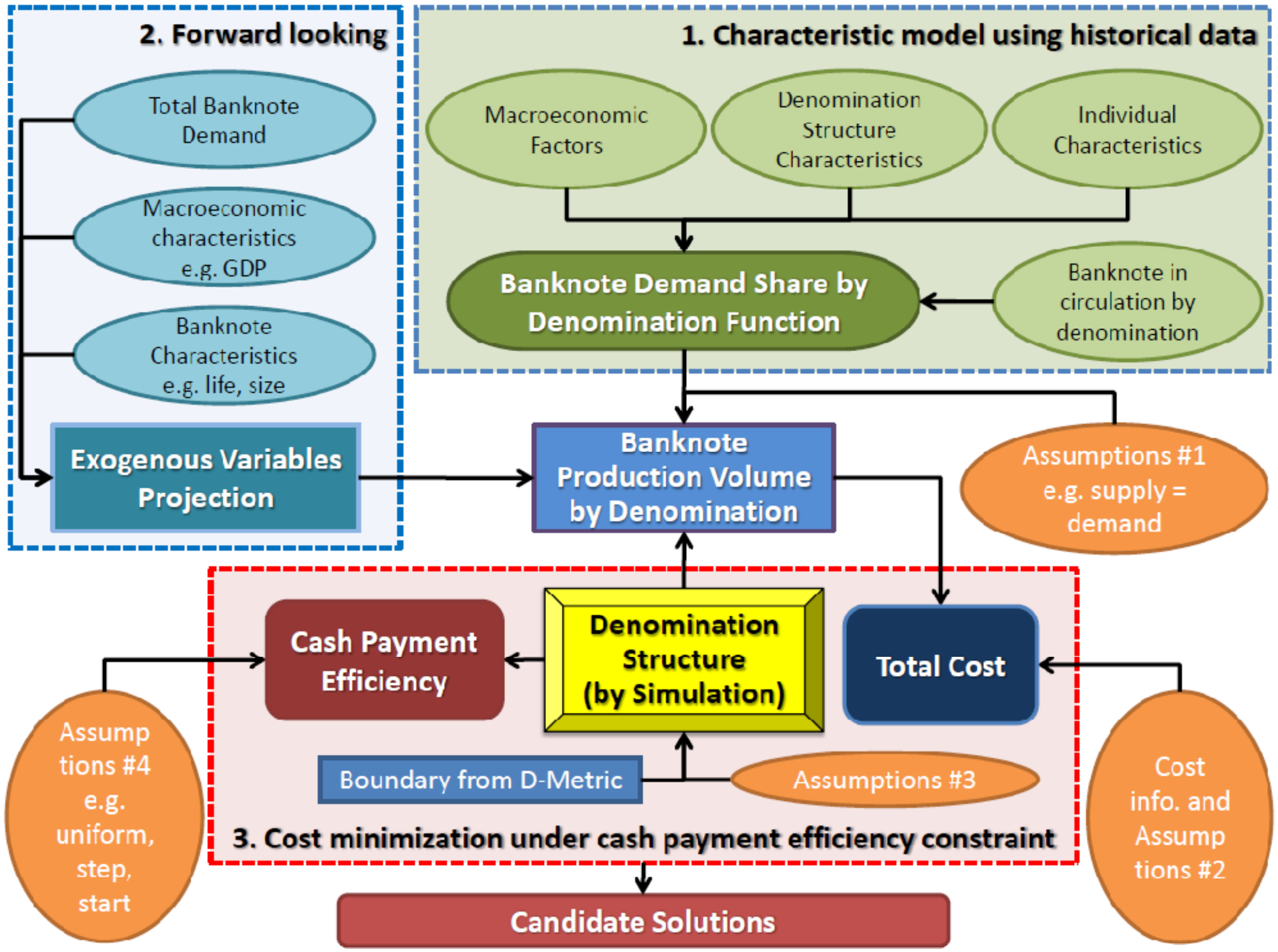


Figure 5.1 Empirical Analysis Methodology

5.2 Banknote Demand by Denomination

According to the characteristic model discussed in chapter 3, it is applied in this research to estimate the banknote demand share by denomination. The details are as follows:

5.2.1 Model, Functional Form, Assumptions, and Data

The model is composed of many equations, and each equation belongs to one denomination demand share. The characteristic model is applied to the analysis of the banknote demand share function by denomination in the case of Thailand. With this approach, the empirical results can be extended to predict the future demand share of various banknote denominations, including denominations which did not exist before (Kohli, 1988: 390).

Based on equation (3.5), the model representing the demand share for banknote denomination n (S_n) is a little bit adjusted; that is, the interaction terms of the common characteristics are the logarithmic term of their face values. This is the idea of the research—to compress the interaction terms and make them not too different, e.g. ln(1000) and ln(20) is less different than 1000 and 20 baht.

The equation of banknote demand share for denomination n is shown in equation (5.1).

$$\boldsymbol{S_n} = \frac{\left(e^{\sum_j (\alpha_j A_{jn}) + \sum_k [\beta_k B_k \cdot \ln(F_n)]}\right) \cdot F_n^{1-\sigma}}{\sum_{i=1}^{I} \left[\left(e^{\sum_j (\alpha_j A_{ji}) + \sum_k [\beta_k B_k \cdot \ln(F_i)]}\right) \cdot F_i^{1-\sigma}\right]} \qquad (5.1)$$

where

S_i = demand (in value) share for banknote denomination i ;

A_{ji} = individual characteristic j for banknote denomination i;

B_k = common characteristic k for all banknote denominations;

F_i = face value of banknote denomination i

α_j = coefficient of individual characteristic j

β_k = coefficient of common characteristic k

σ = elasticity of substitution

In this study, to estimate banknote demand share functions, the annual time series data covering 30 years since 1981 were collected for running the model. There were six banknote denominations in the study period, which are 10, 20, 50, 100, 500, and 1000 baht. Thus there were six demand share equations (i = 1, 2 , 3, 4, 5 and 6). In each equation, the dependent variable was the demand share in value for the specific denomination, while the independent variables were its characteristics, which are categorized into three groups. The first group is composed of the individual characteristics for each denomination, such as size and color. The second group is composed of the attributes of a banknote denomination structure, such as the number of denominations and the transition between coins and banknotes. The last group is composed of relevant macroeconomic factors, such as price level, gross domestic product (GDP), and electronic payment. It can be found that the characteristics in the first group are individual characteristics, while the second and third groups act as common characteristics, in which all denominations have the same characteristics. Table 5.1 shows the examples of the characteristics classified according to three types, which are selected to be the independent variables for the characteristic model later on.

Table 5.1 The Possible Characteristics Affecting the Demand for Banknote Denominations

Own Characteristics	Denomination Structure	Macroeconomic Factors
- Size	- The number of currency denominations	- GDP
- Color	- The transition between coin and banknote	- Price level
- Average spread which is defined as the square root of sum square of the next lower and upper spreads	- The highest banknote value	- E-payment
- The 10^n value i.e. 10, 100 and 1000	- The lowest coin value	- Interest rate
- Counterfeiting risk	- Average spacing factor of overall structure	- Exchange rate
- Small or large-valued denomination group		- Inflation
- ATM		

As the second and third groups, which are denomination structure and macroeconomic variables, are common characteristics, the research introduces the interaction terms by multiplying them by their face value in order to avoid the case of identical impact. As discussed in section 3.2, the interpretations of these factors in the demand share equation are therefore focused on the comparison between the low and high value of banknote denominations. If the variable coefficient is positive, that factor favors high value denomination and vice versa.

It should be remembered that the assumptions behind the model according to the equation (5.1) are:

1) The utility function is CES.

2) The characteristic function or h(•) is in a form of log-lin model.

3) All equations have the same parameter values for each characteristic. This assumption can be assigned if we believe that all characteristics are taken into account and put into the model (Kohli, 1988: 392; Kohli and Morey, 1990: 56).

4) The common factors are multiplied by the interaction terms, which are their face values in logarithmic terms.

In addition, the model is run using "non linear seemingly unrelated regression" or "nlsur" with the STATA program in order to solve the non-linear equations system. Because the demand shares of all denominations are summed to be 1, one equation has to be ignored from the equation system (Kohli, 1988: 394; Kohli and Morey, 1990: 59).

5.2.2 Hypothesis

The explanation of the independent variables in detail, including the sign hypothesis, can be explained as follows[23]:

[23] To interpret the estimators from a specific demand share function, the comparative-static results in section 3.2 are required to help explain the coefficients signs.

1) The First Group – Own Characteristics

(1) The size of banknote

The size of the banknote was measured by rectangular area in cm^2. In general, the size of banknote usually varies according to the value of money so that users can differentiate the money they have to pay and store, e.g. the 1000-baht banknote is bigger than the 20-baht banknote. Therefore, in a psychological sense, people tend to prefer banknotes of larger size, which have greater value. On the other hand, people may be satisfied with banknotes of small size rather than large size banknotes because of their convenience for carrying and use. In sum, size is expected to be either positively or negatively related to demand, which results in an ambiguous hypothesis.

(2) The 10^n value, i.e. 10, 100 and 1000

This variable was taken into the model as a dummy with a unit value if the banknote value was in the power of ten (10^n); otherwise the dummy variable becomes zero. This variable was expected to be positively related to demand, implying that the banknotes with a 10^n value, such as 100 and 1000, are more popular than those with 2 or 5 times 10^n, such as 200 and 500, respectively, in a ceteris paribus condition.

(3) The average spread

The average spread was calculated by the square root of the sum square of the nearest lower and upper spread. For example, in the case of 1-2-5-10-20 denomination structure, if we consider the denomination 10, the lower and upper denominations are 5 and 20, respectively. The lower and upper spreads are then equal to 2 (from 10/5 and 20/10). Thus, the average spread in this case is the sum of their squares, which is equal to $\sqrt{2^2 + 2^2} = 2\sqrt{2}$. The average spread is expected to be positively related to demand because the denomination with a wide average spread implies a low number of its nearly representatives and therefore that denomination will be required in a large amount.

(4) ATM accessibility

ATM accessibility is determined by a dummy, which turns to 1 if the banknotes in that denomination are put into an Automatic Teller Machine (ATM); otherwise the value is zero. ATM accessibility is expected to be positively related to demand, implying that the denomination which is allowed to be put into an ATM is more popular with a higher demand share than the opposite case.

2) The Second Group – Common Characteristics which are the Features of the Denomination Structure

(1) No. of banknote denominations

The number of banknote denominations is expected to favor the high-group denominations because it would raise the possibility of exact payment, which creates less demand for low-group denominations for transactions but increases demand for high-group denominations which serve the purposes of cash as a store of value. The sign of estimator would then be expected to be positive.

(2) Lowest banknote denomination

The lowest banknote denomination is difficult to interpret and is ambiguous. The impact may depend on other factors, such as price level and income.

(3) Highest banknote denomination

The highest banknote denomination is difficult to interpret and is ambiguous. The impact may depend on other factors, such as price level and income.

(4) Average spacing factor

Average space is difficult to interpret and is ambiguous depending on what the series inside is. If the big average space comes from a gap in the low group, the rise and fall in demand share seem to take place in the low-group and high-group banknote denominations, respectively. On the other hand, if the big average space comes from a gap in the high group, the rise and fall in demand share seem to take place in the high-group and low-group banknote, respectively.

3) The Third Group – Common Characteristics which are the Features of the Denomination Structure

(1) Price level

Price level is proxied by the headline consumer price index (CPI), which is expected to favor high-group denominations. The high price level causes high payment amounts and high denominations seem to be needed more for transaction purposes. The sign of the estimator would then be expected to be positive.

(2) E-Payment

E-payment is proxied by credit card spending, which is expected to favor low-group denominations. The growth of e-payments seems to decrease the role of high-group denominations if we believe that e-payments, such as credit cards, mainly replace large amounts of cash payments. The cash is then mainly used for small amounts which need low-group denominations. Therefore, the sign of the estimator would be expected to be negative.

(3) GDP per Capita

The nominal GDP is used to calculate the GDP per capita, which is difficult to interpret and is ambiguous. This factor may affect many things, such as price level and e-payments, which are positively related to GDP per capita through time. The high price level seems to favor high-group denominations because it will be needed more for transaction purposes if prices go up. On the other hand, high e-payment access seems to favor low-group denominations because e-payments such as credit cards are popular for substituting cash usage. Therefore the GDP per capita can favor demand share in both low- and high-group denominations.

(4) Interest rate

Interest rate is proxied by the saving interest rate, which is expected to favor high-group denominations. The high interest rate motivates people to save money rather than use it. In other words, the transaction purpose of cash will be dominated by store of value, which is usually done by high-group denomination banknotes. Therefore, the sign of estimator would be expected to be positive.

4) The Elasticity of Substitution

The elasticity of substitution is expressed by the value of estimated σ. The CES function exhibits a constant elasticity of substitution. Leontief, linear, and Cobb-Douglas functions are special cases of the CES function. That is, in the limit as σ approaches 1, we get the Cobb-Douglas function; as σ approaches positive infinity we get the linear (perfect substitutes) function; and for σ approaching 0, we get the Leontief (perfect complements) function. In the case of banknotes, the CES utility function applied to demand analysis shows the constant elasticity of substitution among denominations. In general, the currency denominations can be substituted for each other; for example, one 100-baht banknote can be replaced by five 20-baht banknotes. However, if user preferences are concerned, it can be found that each denomination may not be perfectly substituted; for example, one 100-baht banknote holds the same value as five 20-baht banknotes but is better in terms of convenience in carrying and use. Therefore, the substitution among each other is expected to be imperfect, i.e. σ value is higher than zero at one specific number but does not approach positive infinity.

In sum, all characteristics including their notations, descriptions, units, expected sign and sources, are shown in Table 5.2.

Table 5.2 Description of Characteristics

Variable	Description	Unit	Expected sign	Source	Remarks
per_capita	GDP per capita	baht	+/-	National Economic and Social Development Board	the proxy of income
i	average saving interest rate	percent	+	Bank of Thailand	the proxy of opportunity cost from handling cash
cpi_h	headline consumer price index (end of period)	index (2007=100)	+	Ministry of commerce, Thailand	the proxy of price level
credit_spend	spending value of credit card	millions of baht	-	Bank of Thailand	the proxy of electronic payment
no_note	number of banknote denominations	denomination	+	Bank of Thailand	
low_note	lowest value of banknote denomination	baht	+/-	Bank of Thailand	
high_note	highest value of banknote denomination	baht	+/-	Bank of Thailand	
space	average spacing factor	baht	+/-	by calculation	a whole series including coin
size	area of rectangular banknote (width*length)	cm^2	+/-	Bank of Thailand	
atm	ATM contains the banknotes with specific denomination	1=yes 0=no	+	by fact	
decimal	the specific denomination is power of ten (10^n) value	1=yes 0=no	+	by fact	
spread	average spread between specific denomination and its neighboring	baht	+	by calculation	

5.2.3 Results

At the beginning of the empirical study on the characteristic model, twelve characteristics were selected and introduced into the model. The numbers of individual and common characteristics were four and eight, respectively. The selected individual characteristics were size, average spread, ATM accessibility, and dummy for the denomination with the power of ten. The selected common characteristics were the number of banknote denominations, the lowest banknote denomination, the highest banknote denomination, the average space in the denomination series, nominal GDP per capita, consumer price index (Headline CPI), e-payment, and interest rate. The result of banknote demand share by denomination analyzed in the initial model is shown in Table 5.3 in the second column. It can be seen that only 5 of the 13 coefficients are significant at the 10% significance level. Moreover, 2 of the 5 estimated equations[24] exhibit a poor fit with 46% and 53% of the coefficient of determination (R^2). Therefore, the research modified the model by dropping some of the independent variables. An attempt was made to select one of the three macroeconomic factors which were related to each other in the same direction, i.e. GDP per capita, e-payment, and price level. E-payment was the first factor to be dropped from the model because it was unclear how to select the appropriate proxy, such as credit card spending, number of credit card accounts, bahtnet, and e-money usage. Moreover, most of the historical data on e-payments have a short series, e.g. 5 years for the e-money series. Between the GDP per capita and price level, GDP per capita was selected to be in the model, although the initial results show that GDP per capita was not significant at the 5-10% significance level while price level was. This is because GDP per capita is more appropriate to be projected in long term rather than price level. Finally, interest rate and ATM accessibility were dropped from the model because they were both insignificant at the 5-10% significance level with unexpected signs. The result of banknote demand share by denomination analyzed with the improved model is shown in Table 5.3 in the third column.

[24] The results show only five share estimated equations to explain six banknote denominations without the equation of the 50-baht denomination. This is due to the fact that the sum of shares has to be 1 and the demand share of 50-baht denomination would be the rest from the shares of the previous five denominations.

Table 5.3 Demand Share by Denomination using Characteristic Model

Dependent : Banknote Demand Share		**Initial Model**	**Improved Model**
Coefficients of Independent Variables:			
per_capita		0.0000164	0.0000015*
		(0.000)	(0.000)
i		-0.967207	
		(0.753)	
cpi_h		0.5587151**	
		(0.292)	
credit_spend		0.0000151	
		(0.000)	
no_note		-5.715187*	0.1318171*
		(1.562)	(0.011)
low_note		1.488579*	0.0089982*
		(0.597)	(0.003)
high_note		-0.011117	-0.000662*
		(0.009)	(0.000)
space		-3.065297	0.2018817*
		(3.822)	(0.026)
size		0.496009*	-0.0144747*
		(0.168)	(0.006)
atm		-0.408484	
		(9.343)	
decimal		0.518937	0.4573497*
		(2.906)	(0.081)
spread		6.611139	0.0754318*
		(n.a.)	(0.023)
Elasticity of Substitution (σ)		0.0548027*	0.9999996
		(0.009)	(n.a.)
Observations		30	30
Parameters		13	9
R-Squared:			
Equation 1	s_v10 (10 baht)	0.4606	0.8949
Equation 2	s_v20 (20 baht)	0.5315	0.9180
Equation 3	s_v100 (100 baht)	0.9039	0.9080
Equation 4	s_v500 (500 baht)	0.9765	0.9030
Equation 5	s_v1000 (1000 baht)	0.9918	0.9170

Note: The figures in parentheses show the standard errors of estimators.
(*) and (**) denotes statistical significances at 5% and 10% significance levels.
The printouts are shown in Appendix E.

As a final result, the estimated banknote demand share equation (with eight characteristics) is

$$s_i = \frac{(e^{\sum_{j=1}^{8} \beta_j a_{ji}})^{\sigma} \cdot F_i^{1-\sigma}}{\sum_{i=1}^{I} (e^{\sum_{j=1}^{8} \beta_j a_{ji}})^{\sigma} \cdot F_i^{1-\sigma}} \qquad ; i = 1,2,3, \dots, I \qquad (5.2)$$

where the estimated parameters are shown in Table 5.4,

Table 5.4 The Estimated Parameters of Eight Characteristics

Characteristics			Estimated Parameters	
Macroeconomic factors	a_1	per_capita times denomination face value (in logarithmic term)	$\hat{\beta}_1$	0.0000015*
Overall Structure	a_2	no_note times denomination face value (in logarithmic term)	$\hat{\beta}_2$	0.1318171*
	a_3	low_note times denomination face value (in logarithmic term)	$\hat{\beta}_3$	0.0089982*
	a_4	high_note times denomination face value (in logarithmic term)	$\hat{\beta}_4$	-0.000662*
	a_5	space times denomination face value (in logarithmic term)	$\hat{\beta}_5$	0.2018817*
Individual Characteristics	a_6	size	$\hat{\beta}_6$	-0.0144747*
	a_7	decimal	$\hat{\beta}_7$	0.4573497*
	a_8	spread	$\hat{\beta}_8$	0.0754318*
Elasticity of Substitution			$\hat{\sigma}$	0.9999996

Note: (*) denotes statistical significance at the 5% significance level.

Note that the estimated demand equation can be applied to all banknote denominations, including the denominations which have not existed. This is due to the assumption that all characteristics are concerned and taken into the model. Determining the statistical significances and coefficient signs, each characteristic can be interpreted as follows:

1) Area

The size of the banknote can explain the demand share in the negative direction at 5% significance, which implies that people prefer smaller size banknotes rather than larger ones. Consumers seem to need banknotes of a compact dimension for convenience in use and for keeping in their wallets or purses rather than needing large dimension banknotes to add value in a psychological sense.

2) Decimal

The decimal refers to the dummy, indicating whether the specific denomination is in the form of $10^{n,}$ such as 1, 10, 100, and 1000. The result shows that the decimal factor can explain demand share in a positive direction at 5% significance. In other words, with the denomination in the form of 10^n, the demand share increases. This is in line with the hypothesis in the sense that the denominations in the form of 10^n are convenient to use, count, and calculate.

3) Spread

The average spread explains the demand share in a positive direction at 5% significance. The denomination with a higher average spread shows a larger gap between itself and the nearest denominations (above and below), which implies less similar representatives and therefore the demand for that denomination will increase.

4) Number of banknote denominations

The number of banknotes can explain demand share at 5% significance. The positive sign shows that this factor favors the high-group denomination, implying that when the number of banknotes increases, the banknote demand shares tend to be greater for high-value denominations but smaller for low-value denominations. Larger-number banknote denominations allow people to have various denominations for transaction purposes with a higher chance of making an exact payment, so the demand share seems to be re-allocated from especially the low-group banknote denominations, which are mainly applied for means of payment, to the high-group banknote denominations, which are mainly applied to being a store of value.

5) Low note

The lowest value of a banknote can explain demand share at 5% significance. The positive sign shows that this factor favors the high-group denominations, implying that when the value of low the denomination increases, the banknote demand shares tend to be greater for high-value denominations but smaller for low-value denominations. The rise of the lowest value banknote will transfer the role of money for transaction purposes from banknotes to coins with the highest denomination and increase the store of value purpose to the banknote. For this reason, the demand shares increase for high-value banknotes but decrease for low-value banknotes. Because of the fact that the denomination series increases in geometric progression near to 100%, for example, from 1 to 2 (100%), 2 to 5 (150%), 5 to 10 (100%) and 10 – 20 (100%), the values of denominations rapidly grow from lowest to highest. It can be seen that, in the case of the structure 10 – 20 – 50 – 100 – 200 – 500 – 1000, the first and seventh denominations are 10 and 1000, respectively. In this research, the term "value effect" is defined to explain this phenomenon of currency demand shares by value, which favors high denomination due to the rapid growth from the geometric progression discussed earlier. Therefore, the larger and smaller demand shares of high- and low-denomination banknotes when the lowest-denomination banknote increases may also come from the "value effect."

6) High note

The highest value of a banknote can explain demand share at 5% significance. The negative sign shows that this factor favors the low-group denomination, implying that when the highest banknote denomination increases, the banknote demand shares tend to be smaller for high-value denominations but greater for low-value denominations. The increase in the highest banknote denomination will cause the high-group denominations to lose their demand shares to other denominations because that specific highest denomination can be substituted and seems to play the same role of money as the store of value.

7) Spacing

The average space of all banknote denominations can explain demand share at 5% significance. The positive sign shows that this factor favors the high-group denomination, implying that when average space increases, the banknote demand shares tend to be greater for high-value denominations but smaller for low-value denominations. In fact, the impact of this factor is ambiguous depending on what the series inside is (in the case of ceteris paribus, that means the other factors such as number of banknote denominations are fixed). If the large average space comes from a gap in the low group, the rise and fall in demand share seem to take place in the low-group and high-group banknotes, respectively. On the other hand, if the large average space comes from a gap in the high group, the rise and fall in demand share seem to take place in the high-group and low-group banknotes, respectively. The empirical result shows the latter case, which may be due to the historical denomination structure of Thailand generating a gap at the end of the banknote series (without 200 baht), which corresponds to the case of the gap in the high-group banknotes, as mentioned earlier. Moreover, the "value effect" may also help explain this phenomenon in the sense that high-group denominations tend to capture more demand share than low-group denominations.

8) GDP per capita

The GDP per capita can explain demand share at 5% significance in line with the hypothesis. The positive sign shows that this factor favors the high-group denomination, implying that when GDP per capita increases, the banknote demand shares tend to be greater for high-value denominations but smaller for low-value denominations. As discussed earlier, the impacts of GDP per capita on banknote demand share is ambiguous concerning whether it favors high- or low-group denominations. The empirical result shows that high GDP per capita favors high-group denominations, implying that income and price level are better transmissions for explaining these impacts rather than e-payment, which is expected to replace cash usage. As income increases, price level tends to increase and the demand for banknotes also increases, especially for the high denominations according to the empirical results. In other words, the high-group denominations can capture high demand shares, which emphasizes that they are not replaced or substituted by e-payment. It can therefore be implied that until now, the growing e-payment, which in line with the growth in the GDP per capita, does not yet play a major role in transactions, and cash is still the main means of payment in Thailand.

9) The elasticity of substitution

The estimated σ is a positive number, which implies that the denominations are imperfectly substituted for each other (perfectly if σ approaches positive infinity). Moreover, the approximate value is 1, implying that the utility gained from using banknotes under the given denomination structure can be explained by Cobb-Douglas function.

It can be found that only signs and statistical significances are two points we focus on. The magnitudes of estimated coefficients are less interesting because it is difficult to interpret the coefficients of the share equations, which are in the form of complicated fractions. Moreover, we allow many related variables to be taken into account, such as the number of denominations, average spread, average space, and highest banknote denominations, which are not independent of each other. There are two reasons: 1. we focus on a prediction of the demand share rather than an investigation of the factors impacting demand share. Thus, all significant factors should be in the model if they can help explain the demand share. 2. We need a model with the equal coefficients for all share equations, which can be assumed only if all factors are put into the model (Kohli, 1988: 392; Kohli and Morey, 1990: 56).

5.3 The Projection of Other Components

In order to carry out a cost minimization by comparing all possible alternative structures, the estimated characteristics and related components are needed to determine the banknote production plan. The required elements can be divided into two groups: banknote properties and macroeconomic variables. Note that this study does not focus on forecasting technique. Acceptable and simple methods are applied, such as linear regression, with some additional assumptions and currency issuing authorities' judgments.

5.3.1 Banknote Properties by Denomination

1) Size of Banknote

The size of banknote is defined as the area of the rectangle by the product of its width and length. The historical data show an equal width for all banknotes and a linearly incremental change along the increase of denomination values. The estimates of banknotes sizes in this section therefore apply the same rules, as shown in Table 5.5. However, it can be observed that the incremental change of new denominations will be reduced by half to control the maximum length of highest-denomination banknotes. Moreover, the estimates are assumed to be constant during the study period (2011-2030).

Table 5.5 The Estimation of Banknote Sizes by Denomination in 2011-2030

	Denomination Value	Width (cm)	Length (cm)	Area (cm^2)
Existing denominations	20	7.2	13.8	99.4
	50	7.2	14.4	103.7
	100	7.2	15.0	108.0
	500	7.2	15.6	112.3
	1000	7.2	16.2	116.6
New denominations	200	7.2	15.3	110.2
	2000	7.2	16.5	118.8

Note: The figures are estimated by Banknote Issuing Authority.

2) Lifespan of Each Banknote Denomination

The lifespan of banknote denominations is estimated by the banknote issuing authority; the Note Printing Works, Bank of Thailand. The figures are shown in Table 5.6. It can be seen that the banknote denomination value and its lifespan are positive correlated with each other. This is because the banknotes with a low value seem to be more frequently circulated for transactions rather than the higher denominations. The estimations applied a simple linear relationship but are separated into two groups: low value and high value. The low-value group is composed of the banknote denominations that are less than 200 baht. Denominations above 200 baht are classified in the high-value group. However, the lifespan of the 200-baht banknote, which has never existed in circulation, is the average of the lifespan of its neighboring banknotes (100 and 500 baht) Moreover, the estimates are assumed to be constant during study the period (2011-2030).

Table 5.6 The Estimation of Banknote Lifespan by Denomination in 2011-2030

Denomination Value	20	50	100	200	500	1000	2000
Lifespan (months)	14	15	16	18	20	21	22

Note: The figures are estimated by Banknote Issuing Authority.

5.3.2 Macroeconomic Variables

1) Nominal GDP (NGDP)

Actually, the long-term projection of the GDP might be imprecise due to the fluctuation of exogenous factors from various sectors, such as the trade sector, the public sector, and the real sector. Consequently, we avoid applying a complicated macroeconomic model to estimate the uncertainly in the long-run prediction. The research applies historical data and time trend to forecast the NGDP for twenty years. The figures are projected in three scenarios: low, high, and base cases, with constant growth rates equal to 6%, 9.5%, and 6.9%, respectively as shown in Figure 5.2. In this research, however, the base case is selected and applied to the numerical analysis for Thailand.

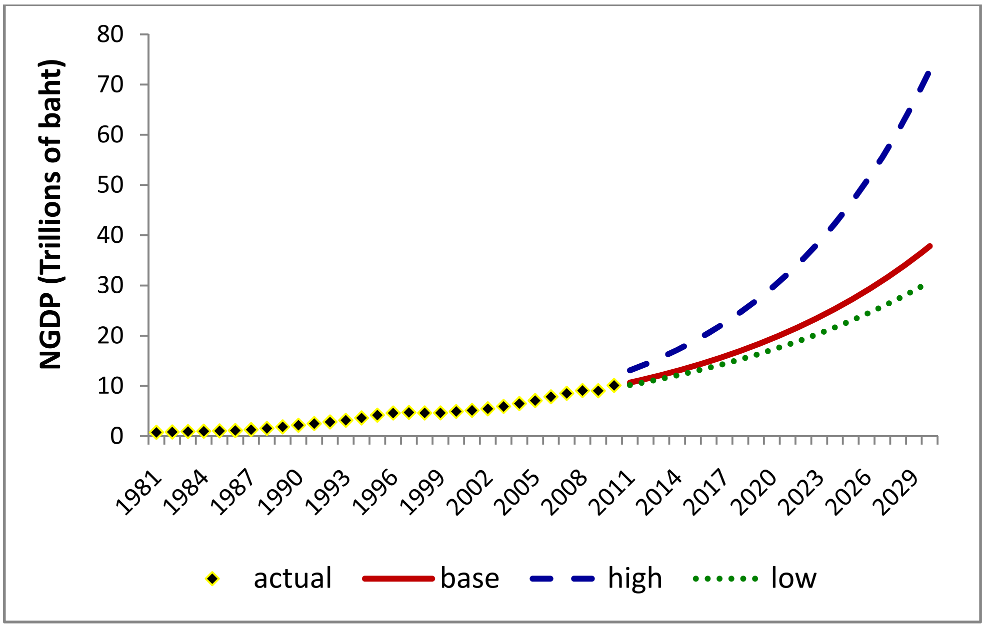


Figure 5.2 The Projection of Thai Nominal GDP for 2011-2030

2) Population

The forecast for the population was prepared by the National Economic and Social Development Board of Thailand (NESDB). The technique in demography called the component method, was applied to estimate the figures. The projection was divided into two scenarios according to the assumption regarding the fertility rates: high and medium levels, as shown in Figure 5.3. In this research, however, the medium scenario as selected and applied to the numerical analysis for Thailand.

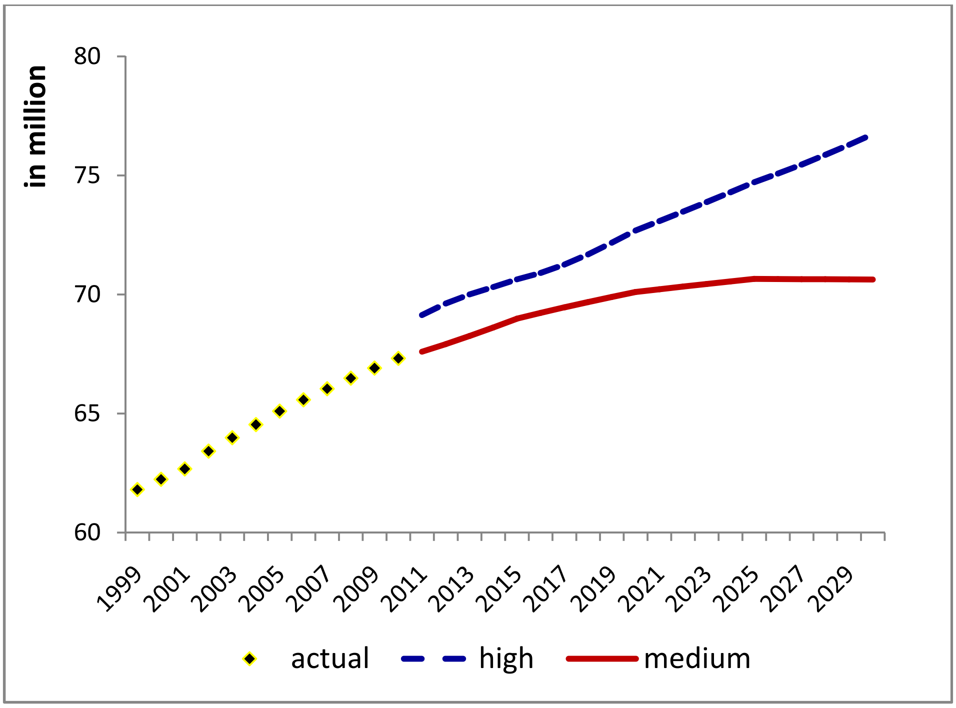


Figure 5.3 The Projection of the Thai Population for 2011-2030

Source: Office of the National Economic and Social Development Board (NESDB): 2007

3) NGDP per Capita

According to the projection for the NGDP and population in 5.3.2.1 – 5.3.2.2, the NGDP per capita can be forecasted by simple calculation, as shown in Table 5.7.

Table 5.7 The Projection of Thai Nominal GDP per Capita during 2011-2030

	Year	Nominal GDP per Capita (Baht)
Actual	2006	119634
	2007	129089
	2008	136586
	2009	135145
	2010	150090
Projection	2011	157602
	2012	167689
	2013	178361
	2014	189665
	2015	201658
	2016	214808
	2017	228852
	2018	243850
	2019	259865
	2020	276966
	2021	295573
	2022	315448
	2023	336676
	2024	359350
	2025	383563
	2026	410049
	2027	438356
	2028	468612
	2029	500956
	2030	535532

4) Total Banknote Supply

Total banknote supply is assumed to be tied to the NGP. It can be seen from Figure 5.4 that total banknote supply and NGDP move along with each other with increasing trends over time. If total banknote supply is linearly regressed on the NGDP, the estimated model yields a high coefficient of determination (R^2) which is equal to 0.968, as shown in Figure 5.5. The projection of total banknote demand in this research was therefore prepared by applying the following estimated function:

$$\text{Total banknote demand} = 0.112(\text{NGDP}) - 75010 \qquad (5.3)$$

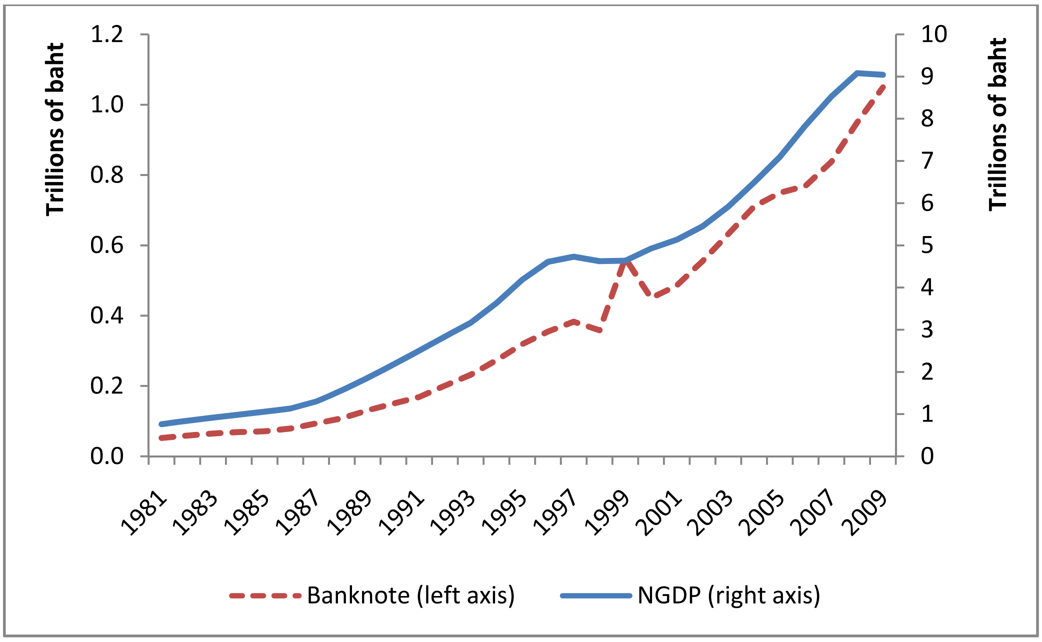


Figure 5.4 The Co-Movement between Total Banknote Supply and NGDP

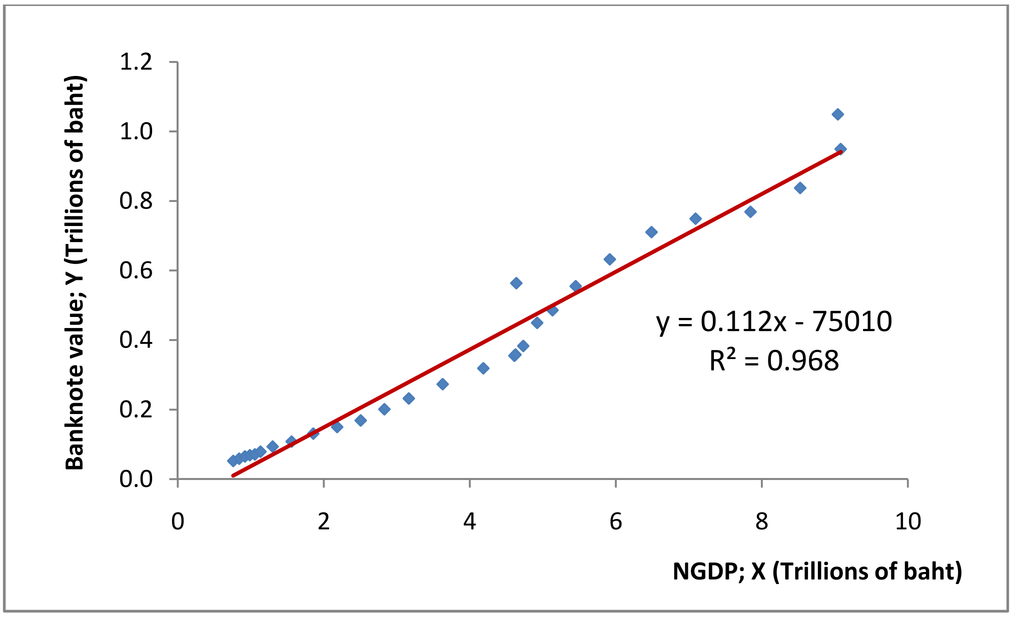


Figure 5.5 The Linear Relationship between Total Banknote Supply and NGDP

5.4 Simulation of Denomination Structures

According to the model shown in section 3.6 and the prediction of relevant components from section 5.3, the optimization problem can be solved by simulation of possible denomination structures and determination of total costs and cash payment efficiencies for all scenarios. The solution would be the structure which gives the minimum total cost under cash payment efficiency constraints.

5.4.1 Model Specification

The model would be set up in two themes: static and dynamic analysis.

1) Static Analysis

The static analysis was prepared for the first period (2011-2015) to check whether the banknote denomination structure was optimal from the cost and efficiency perspective. According to section 3.5 in chapter 3,

Minimize total cost =

$$\sum_{i=1}^{I}\left[\frac{12(e^{\sum_j \alpha_j A_{ji}+\sum_k \beta_k B_k F_i})\cdot F_i^{1-\sigma}\cdot M\cdot VC_i}{L_i\cdot D_i\cdot \sum_{i=1}^{I}[(e^{\sum_j \alpha_j A_{ji}+\sum_k \beta_k B_k F_i})\cdot F_i^{1-\sigma}]}\right] + FC_{1i} + FC_{2i}\cdot {x_i}^2 + FC_{3i}\cdot x_{i(-1)}^2 - (FC_{2i}+FC_{3i})\cdot x_i\cdot x_{i(-1)} \qquad (5.4)$$

2) Dynamic Analysis

The re-structuring of denominations should be a long-term plan because it takes time for changes in both supply and demand. On the demand side, people have to change some payment styles, e.g. carrying, calculating, and memorizing. On the supply side, there will be a modification of the production and distribution of banknotes, e.g. the printing press and ATM. The optimal solution therefore comes from a multi-period analysis. Because there are some linkages or transitions between periods, such as cost of issuing new denominations, dynamic optimization is required for this study in order to obtain, simultaneously, the set of multi-period solutions. In this study, therefore, the dynamic analysis was prepared for 20 years for the period 2011-2030 in order to look forward to the optimal denomination from the perspective of cost minimization with high cash payment efficiency. The reason that only 20 years was studied in advance was because there will be some new and important factors or some shocks in the long term, such as e-payment, that will affect in a major way the optimal denomination structure. In addition, the 20-year range was divided into 4 periods. Each period covers 5 years, i.e. the first period for 2011-2015, the second period for 2016-2020, the third period for 2021-2025, and the fourth period for 2026-2030. The reason why 5 years were added to be 1 period was because within 5 years, it is quite sure that the denomination structure will not and should not be modified. According to section 3.6 in chapter 3,

Minimize total cost =

$$\sum_{t=1}^{T} \sum_{i=1}^{I} \frac{1}{(1+\beta)^t} \left[\frac{12(e^{\sum_j \alpha_j A_{jit} + \sum_k \beta_k B_{kt} F_{it}}) \cdot F_{it}^{1-\sigma} \cdot M_t \cdot VC_{it}}{L_{it} \cdot D_i \cdot \sum_{i=1}^{I} [(e^{\sum_j \alpha_j A_{jit} + \sum_k \beta_k B_{kt} F_{it}}) \cdot F_{it}^{1-\sigma}]}\right] + FC_{1it} + FC_{2it} \cdot {x_{it}}^2 + FC_{3it} \cdot x_{i,t-1}^2 - (FC_{2it} + FC_{3it}) \cdot x_{i,t} \cdot x_{i,t-1} \quad (5.5)$$

5.4.2 Denomination Structure Boundary

In this research, the denomination structure boundary was controlled and earlier given in the optimization process. As stated in the scope of the research, this study focuses on the series inside the given boundary. This section therefore discusses the boundary, which is composed of the lowest-highest denominations and transition between coins and banknotes. Actually, finding an appropriate boundary involves many factors, e.g. inflation, consumer behavior, and psychological impact. The traditional and popular method is the D-Metric model, but there are some limitations to that approach, for example, the absence of other factors which should be taken into consideration, such as the costs associated with economic agents and users' preferences, including payment habit. The more precise method might be the cost-benefit analysis (CBA) or cost-effectiveness analysis (CEA), but accurate analyses are much more complicated. The difficulties of the CBA and CEA are found in how to evaluate all relevant costs and benefits without any double counting, especially when the indirect costs and benefits have to be included in an analysis. Moreover, the valuation technique may be needed to measure some intangible costs and benefits, such as cost of complicated mental calculation and the benefit of a simple structure, which is easy memorize. The researcher then decided to scope the numerical analysis and not to study the boundary in detail; the real practice, however, together with the D-Metric model determination, will be referred to.

In order to apply the D-Metric model, the nominal GDP per capita per day in baht was used to represent an average day's pay (D in D-Metric diagram[25]). The projection of the nominal GDP per capita is shown in Table 5.7. The results of D/500 – 5D until 2030 are illustrated in Appendix F. Together with the judgment of banknote authorities, the projection of the boundary of the currency denomination structure would be controlled, as shown in Table 5.8. It can be seen that the boundary was simultaneously re-structured for three components in 2021. The 0.25-baht coin has to be eliminated from circulation together with the introduction of the new 2000-baht banknote into the structure. Moreover, the 20-baht banknote will be replaced by 20-baht coins at the same time.

Table 5.8 The Controlled Boundary of Denomination Structure until 2030

Period	Coin Denomination (baht)		Banknote Denomination (baht)	
	Lowest	Highest	Lowest	Highest
2011	0.25	10	20	1000
2011-2015	0.25	10	20	1000
2016-2020	0.25	10	20	1000
2021-2025	0.50	20	50	2000
2026-2030	0.50	20	50	2000

Note: The boundary is estimated from D-Metric Model for Thailand[26] with some Banknote Issuing Authority's Judgment.

[25] See Appendix D

[26] See Appendix F

5.4.3 Cost Information

According to the total cost structure presented in chapter 3, total cost is composed of fixed and variable costs. Fixed cost does not include administration and other costs, which are fixed for whatever the denomination structure is. Therefore the total cost to be optimized does not mean the actual overall costs. In this research, costs details according to banknote denomination are collected and estimated by the Note Printing Works as shown in Table 5.9. It can be observed that average variable costs are assumed based on past information to be linearly raised by 0.25 baht, along with the increasing of banknote denominations. On the other hand, fixed costs are assumed to be equal for all denominations. Three kinds of fixed cost are also estimated from determining the past information from both central bank and commercial banks. For simplicity, cost details by denomination are assumed not to change over time. This might be reasonable if it is supposed that the likely high cost in the future from increasing price level is offset by production technology, which would result in lower production cost. Moreover, a 5% discount rate is taken into the account in dynamic analysis because 1 baht today is more valuable than in the future.

Table 5.9 The Fixed and Average Variable Costs by Denomination during 2011-2030

Banknote Denomination (Baht)	Average Variable (Unit) Cost (Baht/Note)	Fixed Cost		
		Re-Structuring Fixed cost		Annually Routine Fixed Cost per Denomination (Millions of Baht)
		for New Banknote Introduction (Millions of Baht)	for Old Banknote Elimination (Millions of Baht)	
20	0.50	130	60	900
50	0.75	130	60	900
100	1.00	130	60	900
200	1.25	130	60	900
500	1.50	130	60	900
1,000	1.75	130	60	900
2,000	2.00	130	60	900

Note: The figures are estimated by Banknote Issuing Authority.

5.4.4 Cash Payment Efficiency

According to section 3.5 in chapter 3 regarding the cash payment efficiency analysis, there are two aspects in measuring cash payment efficiency: the average number of minimum tokens (tok) and the average number of efficient payment schemes (sch). To calculate tok and sch, the cash payment profile is required to weigh the payment efficiency among all payment amounts. In this research, the uniform distribution of cash payments is assumed for simplicity. Bouhdaoui, Bounie, and Francois (2009: 1) showed in their study that the average number of minimum tokens (tok) obtained from the observed payment amounts distribution is around two times less than that obtained when it is assumed to be uniformly distributed. However, this is not significant for the present research because tok is applied for cash payment efficiency comparison purposes, not for absolute value analysis. Although the cash payment distribution is assumed to be uniform, the payment profile boundary is still needed for calculation. The profile boundary would be a guideline for start-step-end, as discussed in chapter 4. In this research, the cash payment profile boundaries are separated into two scenarios: in 2011-2020 and in 2021-2030. Each scenario gives the set of payment efficiency for various denomination structures in both aspects, as shown in Table 5.10 – 5.11. It can be seen that the full-series structures in both scenarios have six banknote denominations. If the highest and lowest denominations (transition between coins and banknotes) are given and fixed, as we earlier assumed, there will be sixteen possible structures for candidates[27]. As discussed in the beginning of chapter 3, the payment efficiency has the role of being a constraint, which is not directly included in an objective function. However, only the first aspect (tok) of concern and will be a part of the constraint. This is because the definition of tok is directly in line with the concept of efficiency according to the principle of least effort. On the other hand, the second aspect (sch) is an additional scheme subsequently defined. In this research, sch was considered for checking the performances of the structures regardless of the optimization process.

[27] The coin denomination structure is assumed to be a full series over the study periods.

Table 5.10 Cash Payment Efficiency Determination of all Possible Scenarios during 2011-2020

Full Series:	**Coin**	**Banknote**
	0.25-0.50-1-2-5-10	20-50-100-200-500-1000

Payment Profile Boundary	**Start**	**Step**	**End**	**No. of Cash Payment Amounts**
	1	1	2000	2000

Structure Description	**No. of Denominations**	**Average Space**	**No.of Minimum Tokens**			**No. of Efficient Payment Schemes**		
			Min	**Max**	**Average**	**Min**	**Max**	**Average**
without 50 100 200 500	8	8.93	1	28	14.92	1	3	1.55
without 100 200 500	9	4.38	1	14	7.93	1	6	1.94
without 50 200 500	9	3.44	1	10	6.08	1	5	1.72
without 50 100 500	9	3.44	1	10	6.09	1	6	1.82
without 50 100 200	9	4.94	1	16	8.92	1	5	1.57
without 200 500	10	3.00	1	10	5.70	1	9	1.87
without 100 500	10	2.67	1	8	5.04	1	12	2.15
without 100 200	10	3.00	1	10	5.68	1	9	2.01
without 50 500	10	2.72	1	8	5.09	1	6	1.79
without 50 200	10	2.72	1	8	5.08	1	8	1.86
without 50 100	10	3.00	1	8	5.17	1	5	1.60
without 500	11	2.40	1	8	4.79	1	15	2.47
without 200	11	2.40	1	8	4.70	1	9	1.96
without 100	11	2.35	1	7	4.54	1	9	2.08
without 50	11	2.40	1	7	4.69	1	8	1.93
full	12	2.14	1	7	4.39	1	18	2.55

Table 5.11 Cash Payment Efficiency Determination of all Possible Scenarios during 2021-2030

Full Series:	**Coin**	**Banknote**
	0.50-1-2-5-10-20	50-100-200-500-1000-2000

Payment Profile Boundary	**Start**	**Step**	**End**	**No. of Cash Payment Amounts**
	5	5	10000	2000

Structure Description	**No. of Denominations**	**Average Space**	**No.of Minimum Tokens**			**No. of Efficient Payment Schemes**		
			Min	**Max**	**Average**	**Min**	**Max**	**Average**
without 100 200 500 1000	8	7.57	1	26	13.64	1	4	1.59
without 200 500 1000	9	4.38	1	17	8.91	1	6	1.49
without 100 500 1000	9	3.38	1	13	7.00	1	6	1.56
without 100 200 1000	9	3.38	1	13	6.99	1	6	1.61
without 100 200 500	9	4.38	1	17	8.89	1	6	1.62
without 500 1000	10	3.00	1	12	6.75	1	6	1.80
without 200 1000	10	2.67	1	11	6.01	1	6	1.58
without 200 500	10	3.00	1	12	6.66	1	6	1.52
without 100 1000	10	2.61	1	10	5.83	1	6	1.68
without 100 500	10	2.67	1	11	6.00	1	8	1.67
without 100 200	10	3.00	1	12	6.74	1	8	1.96
without 1000	11	2.35	1	10	5.70	1	12	2.19
without 500	11	2.40	1	10	5.75	1	10	1.91
without 200	11	2.40	1	10	5.76	1	8	1.91
without 100	11	2.35	1	10	5.58	1	10	1.90
full	12	2.14	1	10	5.45	1	18	2.58

5.4.5 Static Analysis

This section focuses on the static analysis, which determines the denomination structure during the initial period (2011-2015). The purpose is to examine whether the banknote denomination structure at present is optimal from the cost and efficiency perspective.

1) Possible Scenarios

From a static viewpoint, we control the boundary by assuming the current boundary to be fixed for all scenarios. They are composed of 0.25-baht coins and 1000-baht banknotes as the lowest and highest denominations, respectively. Furthermore, the transition between coins and banknotes is fixed to be the 20-baht banknote. However, there are some additional assumptions used in identifying all possible scenarios. First, only the 1-2-5 structure is of concern because it compromises between the most efficient structure according to the principle of least effort and the compatibility with the decimal currency system. Moreover, most countries around the world widely apply this structure in their currency system. Second, the denominations with power of ten (10^n) values, e.g. 10, 100 and 1000, are always in the series. This is the assumption assigned according to the real practice, in which the denominations with power of ten (10^n) values are popular due to their ease of being used, counted and calculated. Finally, all coin denominations are in the structure since only the banknote denomination structure is focused on in this research. Therefore, all possible scenarios for static analysis are shown in Table 5.12.

Table 5.12 Possible Scenarios of Denomination Structures for Static Analysis

Code	Denomination Structure		Description
	Coin	Banknote	
1	0.25-0.50-1-2-5-10	20-100-1000	without 50 200 500
2	0.25-0.50-1-2-5-10	20-100-500-1000	without 50 200
3	0.25-0.50-1-2-5-10	20-100-200-1000	without 50 500
4	0.25-0.50-1-2-5-10	20-50-100-1000	without 200 500
5	0.25-0.50-1-2-5-10	20-100-200-500-1000	without 50
6	0.25-0.50-1-2-5-10	20-50-100-500-1000	without 200 (at present)
7	0.25-0.50-1-2-5-10	20-50-100-200-1000	without 500
8	0.25-0.50-1-2-5-10	20-50-100-200-500-1000	full series

2) Static Analysis Result

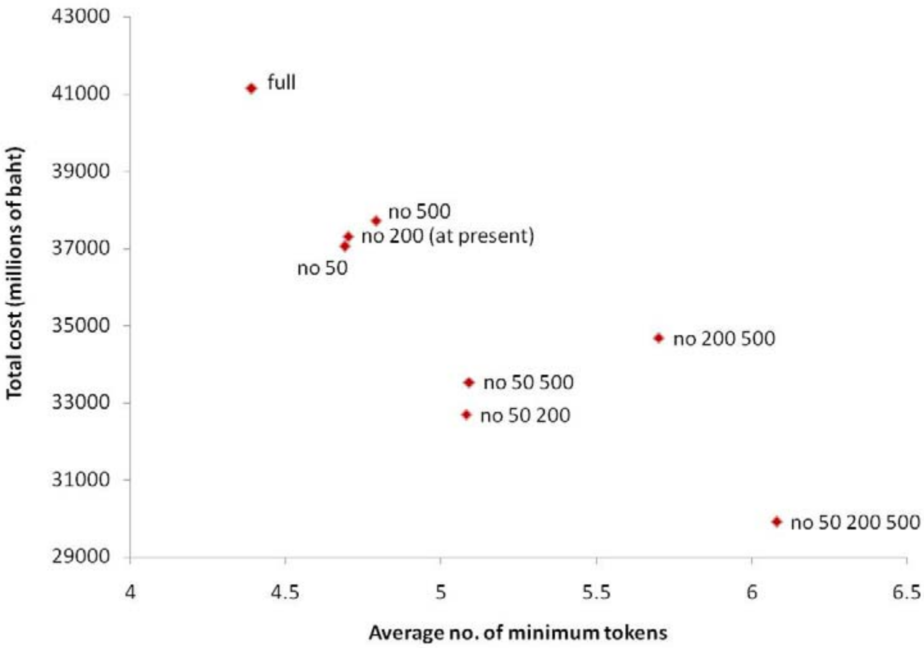
According to Table 5.12, there are eight scenarios to be candidates for optimal solutions. The structure details, including costs and payment efficiencies, are summarized in Table 5.13. It can be observed that the structure with a full series yields the highest payment efficiency in both aspects: average number of tokens = 4.39 (smaller value is more efficient) and average number of efficient schemes = 2.55 (larger value is more efficient). It is also comprises the maximum number of denominations (6) with the lowest average space (2.14). Moreover, this structure will generate the minimum amount of production (approximately 14,531 million of notes) during 5-year period (2011-2015). However, it cannot guarantee the minimum total cost. As earlier discussed in chapter 3, the large number of denominations yields a high fixed cost, especially for issuing and managing costs, even though the variable cost would be saved from the lower banknote production in amount. In this case, the full series achieves highest payment efficiency but results in the highest total cost. It can be therefore implied that fixed cost has a major role in total cost determination.

All eight scenarios can be scatter plotted between total cost and average number of minimum tokens, as shown in Figure 5.6. Panel a) shows all points with a downward sloped trend, while panel b) separates the four dominant structures which would be defined as optimal solutions. In addition, the optimal path, which joins all optimal points, is shown to emphasize completely the trade-off phenomenon between cost and payment efficiency along the optimal path. It can be seen that the present currency denomination structure in Thailand (without the 200-baht banknote) is almost located on the optimal path, which implies that the current structure is nearly optimal according to this research. It can be a little bit improved from both the efficiency and cost perspectives if we eliminate the 50-baht denomination from circulation. Other scenarios on the optimal path will cause one perspective to be better off but worse off for the other one. However, in order to plan for the re-structure in the future, a dynamic analysis might be more appropriate for the purpose. The decision also depends on policy makers concerning how they weigh the significance of cost and efficiency. This will be later discussed in section 5.5.

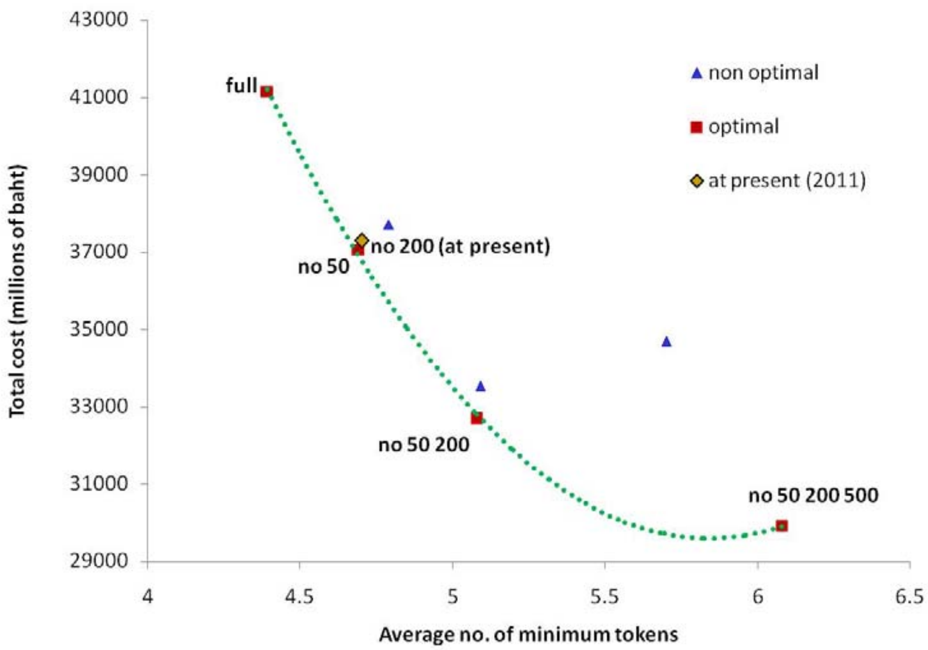
Table 5.13 The Summary of Static Analysis Simulation

Code	Structure	Description	Action	Production 2011 - 2015 (Millions of Baht)	Total Cost 2011 - 2015 (Millions of Baht)	Cash Payment Efficiency		No. of Banknote Denominations	Average Space (Whole Series Including Coins)
						Average No.of Minimum Tokens	Average No. of Efficient Payment Schemes		
1	20, 100, 1000	without 50 200 500	eliminate 50, 500	20127	29914	6.08	1.72	3	3.44
2	20, 100, 500, 1000	without 50 200	eliminate 50	16333	32697	5.08	1.86	4	2.72
3	20, 100, 200, 1000	without 50 500	add 200 eliminate 50, 500	17022	33533	5.09	1.79	4	2.72
4	20, 50, 100, 1000	without 200 500	eliminate 500	20548	34684	5.70	1.87	4	3.00
5	20, 100, 200, 500, 1000	without 50	add 200 eliminate 50	15061	37052	4.69	1.93	5	2.40
6	20, 50, 100, 500, 1000	without 200 (at present)	hold	16128	37103	4.70	1.96	5	2.40
7	20, 50, 100, 200, 1000	without 500	add 200 eliminate 500	16588	37716	4.79	2.47	5	2.40
8	20, 50, 100, 200, 500, 1000	full series	add 200	14531	41140	4.39	2.55	6	2.14

Note: Current denomination structure is 20, 50, 100, 500 and 1000.



a) Illustrated by Points



b) Illustrated by Points and Optimal Path

Figure 5.6 The Relationship between Total Cost and Average Number of Minimum Tokens (Static Analysis)

5.4.6 Dynamic Analysis

This section focuses on the dynamic analysis, which looks toward the optimal denomination structure from the cost and efficiency perspective in order to prepare the re-structure planning for 20 years (2011-2030). Twenty years are divided into 4 periods, with 5 years per each, i.e. 2011-2015, 2016-2020, 2021-2025, and 2026-2030.

1) Possible Scenarios

Applying the given boundary obtained from Table 5.8 and the three assumptions as in the static analysis, an additional assumption is made for identifying all possible scenarios. The assumption is that the denomination $2*10^n$ and $5*10^n$ will not be missed in the same loop of the series. For example, 20 and 50 must not be skipped from the structure. Another example of the denominations in the same loop is 200 and 500. This assumption comes from the static analysis, which shows the very low payment efficiency from too large average spaces when 200 and 500 are skipped from the series. Therefore, all possible scenarios for the dynamic analysis are shown in Table 5.14. It can be seen that the last assumption can narrow down the number of scenarios, which are the combination of possible scenarios during the four periods. The first and second periods have six scenarios each, while the third and fourth periods have three scenarios each. Hence, there will be 324 scenarios to be simulated in the dynamic analysis section. However, the scenarios in Table 5.14 are more complicated than in the case of static the analysis (Table 5.12). In the dynamic analysis, each scenario is composed of four sub-scenarios represented by a 4-digit code. Each digit denotes a sub-code which corresponds to a particular sub-scenario. For example, scenario code 5432 represents sub-codes 5, 4, 3 and 2 for the 1st, 2nd, 3rd and 4th periods, respectively. According to Table 5.14, the four sub-codes can be interpreted as follows: without 500 in the 1st period, without 200 in the 2nd period, full series in the 3rd period, and without 500 in the 4th period. In addition, the benchmark is the scenario code 4411, which represents the structures in four periods with taking no action except for the re-structuring at 2021 according to the controlled boundary given from D-Metric model and shown in Table 5.8.

Table 5.14 Possible Scenarios of Denomination Structures for Dynamic Analysis

Period	Subcode	Denomination Structure		Description
		Coin	Banknote	
2011-2015	1	0.25-0.50-1-2-5-10	20-100-500-1000	without 50 200
	2	0.25-0.50-1-2-5-10	20-100-200-1000	without 50 500
	3	0.25-0.50-1-2-5-10	20-100-200-500-1000	without 50
	4	0.25-0.50-1-2-5-10	20-50-100-500-1000	without 200 (at present)
	5	0.25-0.50-1-2-5-10	20-50-100-200-1000	without 500
	6	0.25-0.50-1-2-5-10	20-50-100-200-500-1000	full series
2016-2020	1	0.25-0.50-1-2-5-10	20-100-500-1000	without 50 200
	2	0.25-0.50-1-2-5-10	20-100-200-1000	without 50 500
	3	0.25-0.50-1-2-5-10	20-100-200-500-1000	without 50
	4	0.25-0.50-1-2-5-10	20-50-100-500-1000	without 200
	5	0.25-0.50-1-2-5-10	20-50-100-200-1000	without 500
	6	0.25-0.50-1-2-5-10	20-50-100-200-500-1000	full series
2021-2025	1	0.50-1-2-5-10-20	50-100-500-1000-2000	without 200
	2	0.50-1-2-5-10-20	50-100-200-1000-2000	without 500
	3	0.50-1-2-5-10-20	50-100-200-500-1000-2000	full series
2026-2030	1	0.50-1-2-5-10-20	50-100-500-1000-2000	without 200
	2	0.50-1-2-5-10-20	50-100-200-1000-2000	without 500
	3	0.50-1-2-5-10-20	50-100-200-500-1000-2000	full series

2) Dynamic Analysis Result

The 324 scenarios, which are all candidates in the dynamic analysis, and the structure details including costs and payment efficiencies are summarized in Appendix G. All scenarios are scatter plotted between total cost and average number of minimum tokens with the optimal path, as shown in Figure 5.7. Similar to the static analysis, the optimal path is created by connecting all dominant points in the downward sloped direction to guarantee a trade-off condition.

It can be seen that there are 13 points which are optimal and placed on the optimal path, whereas the benchmark (code 4411) is not located on the optimal path, implying that it would be not optimal according to our research. It can be improved in both the efficiency and cost perspectives if the scenario is moved to code 1333 which denotes the case of no 50 and 200 in the 1st period, no 50 in the 2nd period, and a full series in the last two periods. Other scenarios on the optimal path will cause one perspective to be better off but worse off for the other one. However, the decision also depends on the policy makers as to how they weigh the significance of cost and efficiency. The details will be discussed in the next section.

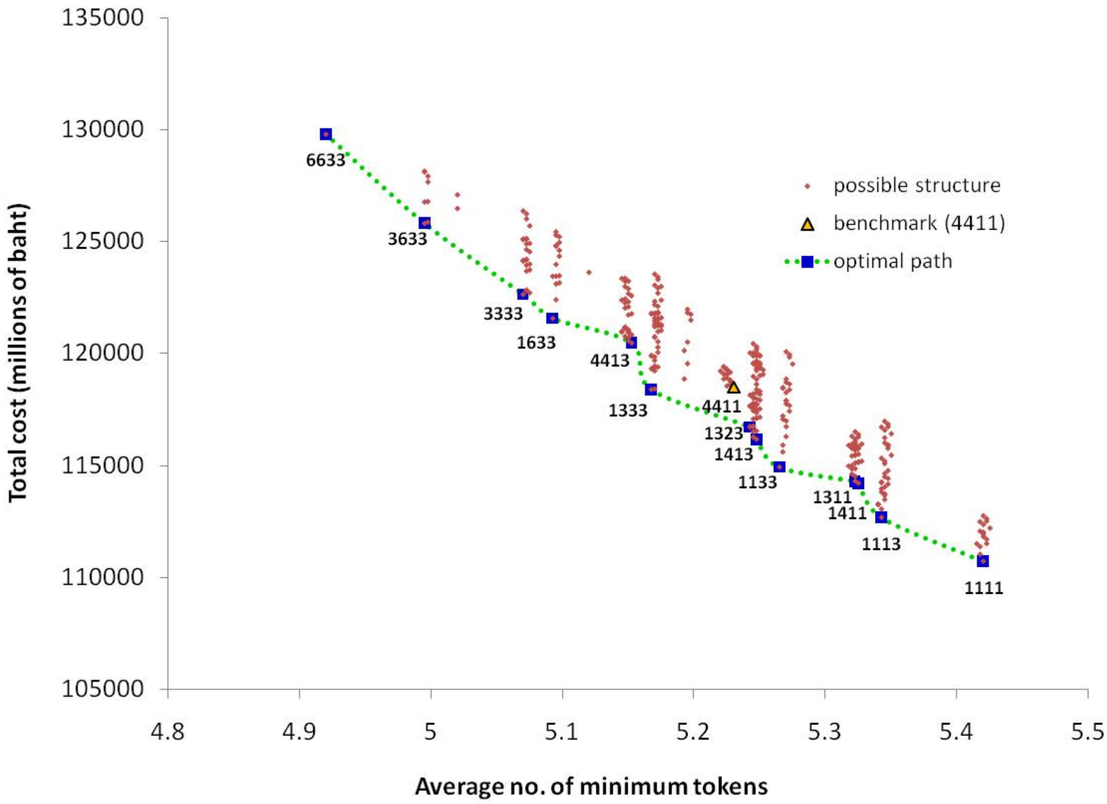


Figure 5.7 The Relationship between Total Cost and Average Number of Minimum Tokens (Dynamic Analysis)

5.5 Optimal Solution According to the Cost and Efficiency Perspectives

According to the simulation results in the last section, the optimal structure is analyzed from two views: static and dynamic.

In this statically numerical study, the four optimal structures on the optimal path in Figure 5.6b, including the current structure, are summarized in Table 5.15-5.16. It can be seen that the existing banknote denomination structure, which is composed of five denominations (20, 50, 100, 500, and 1000 baht) is not optimal. Comparing the four optimal structures, they are a trade-off of each other between cost and efficiency. The ultimate solution would depend on the policy makers' judgment concerning how the significance weights of cost and payment efficiency are assigned. The cost is the perspective from the supply side dealing with the currency issuing authority to favor the denomination which yields the low cost. On the other hand, the efficiency is the perspective from the demand side, which concerns the payers that are assumed to satisfy the denomination structure supporting the payment with minimum tokens. If only payment efficiency is considered (100% weight of efficiency), the structure with a full series will be the ultimately optimal solution. On the other hand, if only cost is of concern (100% weight of cost), the structure with the three denominations (20, 100 and 1000 baht) denominations will be the ultimately optimal solution[28]. The more realistic policy strategy is a compromise between cost and efficiency. The optimization problem in our research is therefore to find the optimal denomination structure which yields minimum cost under the payment efficiency constraints.

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[28] Note that it does not guarantee that the structure with smallest number of denominations always gives the lowest cost. The structure with only one denomination might give high cost from the variable cost even though the fixed issuing cost can be saved. However, one-denomination structures are impossible to be applied in real situations and have to be excluded from our candidate list.

Assume that the payment efficiency constraint regarding the optimization problem is that the payment efficiency must not be worse than the benchmark (current situation). In other words, the average minimum tokens must not be more than the value of the current denomination structure. According to the above constraint, the structure without 50 baht (optimal 2) and full series (optimal 1) are two remaining candidates. Because the structure without 50 baht yields a minimum cost, it is therefore the ultimately optimal solution. Compared to the current structure, the 50-baht banknote has to be eliminated from the circulation together with the introduction of a new 200-baht banknote. This structure can save cost of 0.1% (51 million baht) and increase the payment efficiency in the first aspect, i.e. the average number of minimum tokens decreases by 0.2% (less is preferred to more). As mentioned earlier, that the static analysis is to verify the current denomination structure, it can be found that the current denomination structure is not (but nearly) optimal according to the cost and efficiency perspectives. In addition, if we need the structure with five banknote denominations, the structure without 50-baht denomination seems to be better than another one without the 200-baht denomination, which is the current situation of Thailand. However, the static result and summary do not provide a long-term re-structuring plan for optimal currency denomination. A dynamically numerical study is required for this purpose.

In the dynamic analysis, thirteen optimal structures on the optimal path in Figure 5.7, including the current structure, are summarized in Table 5.17-5.18. It can be seen that the benchmark structure (4411) is not optimal. Structure 6633 will be an ultimately optimal solution if only efficiency is concerned. Conversely, if only cost is concerned, structure 1111 will be the ultimately optimal solution. Suppose the same payment efficiency constraint exists as for the static analysis; that is, the average minimum tokens must not be more than the value of the benchmark structure. There will be six remaining candidates, which are the optimal 1- 6 in Table 5.17. Comparing the six candidates, structure 1333 (optimal 6) would be the ultimate optimal solution since it gives the lowest total cost. With this structure compared to the current situation, the 50-baht banknote has to be removed from circulation now.

Five years later, the 200-baht banknote will be introduced, which is a new denomination. The 50-baht banknote will then be re-entered in 2021 until the end of the study period (2030). This might be due to the greater popularity of the 50-baht banknote because of its higher price level compared to current situation in which the 50-baht banknote should be eliminated from the currency denomination structure. Moreover, as suggested by D-Metric model, 2000-baht banknote will be introduced in 2021 together with the replacement of 20-baht banknotes by 20-baht coins. According to the structure 1333, 0.1% of the total cost (109 million baht) can be saved with higher efficiency in both two aspects. The first aspect is that the average number of minimum tokens decrease by 1.2% (less is preferred to more). The second aspect is that the average efficient payment schemes increase by 15.9% (more is preferred to less). Comparing the ultimately optimal solutions in the static and dynamic analyses, the same suggestion has been proposed: to eliminate the 50-baht banknote. This is reasonable because the 50-baht denomination captures the lowest shares among the other denominations in both value and amount. It might be a cultural preference together with the price level that causes the small share of 50-baht banknotes. Moreover, the neighboring denominations (20 and 100), which are its representatives, have major roles in transactions in present circumstances. However, the static results indicate that the 200-baht denomination is needed now in order to share some of the burden of the 100 and 500-baht banknotes, while the dynamic results propose the introduction of the 200-baht denomination in the next five years.

The ultimately optimal solution could be adjusted according to policy makers to identify the constraints. For example, if the payment efficiency constraint is accepted for the average minimum token, which is the value of the benchmark structure plus or minus a 1% deviation, structure 1133 (optimal 9) will be the ultimately optimal solution. The total cost can be saved by 3%, with the worse efficiency in the first aspect by 0.7%. Nevertheless, the disadvantage of dynamic analysis is the uncertainty of the prediction of relevant factors and corresponding components, such as the GDP and total banknote supply. In order to make a decision regarding re-denomination, we might apply static analysis for short-term preparation together with some additional judgments from policy makers. In this case, the proposed structure from the static problem does not indicate very different performance (a little bit improved). Consequently, the current denomination structure would be the final decision, which is doing nothing now to avoid two major changes (eliminating the 50-baht and adding the 200-baht denominations) in the currency system.

Table 5.15 Descriptions of Optimal Structures according to Static Analysis

Code	Structure	Description	Action
1	20, 100, 1000 (3)	without 50 200 500	eliminate 50, 500
2	20, 100, 500, 1000 (4)	without 50 200	eliminate 50
5	20, 100, 200, 500, 1000 (5)	without 50	add 200, eliminate 50
6	20, 50, 100, 500, 1000 (5)	without 200 (at present)	hold
8	20, 50, 100, 200, 500, 1000 (6)	full series	add 200

Note: The figures in parentheses denote the number of banknote denominations.

Table 5.16 Performances of Optimal Structures according to Static Analysis

Structure			Production Amount (Millions of Notes)			Total Cost (Millions of Baht)			Cash Payment Efficiency				No. of Banknote Denominations	Average Space of Currency Denominations (Including Coins)
									Average No. of Minimum Tokens		Average No. of Efficient Schemes			
Scenario	**Code**	**Description**	**Value**	**Diff**	**%Diff**	**Value**	**Diff**	**%Diff**	**Value**	**%Diff**	**Value**	**%Diff**		
At present (2011) as the benchmark	6	without 200	16128	0	0.0%	37103	0	0.0%	4.70	0.0%	1.96	0.0%	5	2.40
Optimal 1 (efficiency)	8	full series	14531	-1597	-9.9%	41140	4037	10.9%	4.39	-6.6%	2.55	30.6%	6	2.14
Optimal 2	5	without 50	15061	-1067	-6.6%	37052	-51	-0.1%	4.69	-0.2%	1.93	-1.2%	5	2.40
Optimal 3	2	without 50 200	16333	206	1.3%	32697	-4406	-11.9%	5.08	8.1%	1.86	-4.8%	4	2.72
Optimal 4 (cost)	1	without 50 200 500	20127	4000	24.8%	29914	-7189	-19.4%	6.08	29.4%	1.72	-12.1%	3	3.44

Note: A benchmark is a current denomination structure: 20, 50, 100, 500 and 1000.

The difference (diff) with a negative sign shows saving from a benchmark.

Table 5.17 Descriptions of Optimal Structures according to Dynamic Analysis

No.	Code	2011-2015	2016-2020	2021-2025	2026-2030
1	4411 (Benchmark)	**without 200** **20, 50, 100, 500, 1000 (5)**	**without 200** **20, 50, 100, 500, 1000 (5)**	**without 200** **50, 100, 500, 1000, 2000 (5)**	**without 200** **50, 100, 500, 1000, 2000 (5)**
		hold	hold	add 2000 eliminate 20	hold
2	6633	**full series** **20, 50, 100, 200, 500, 1000 (6)**	**full series** **20, 50, 100, 200, 500, 1000 (6)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**
		add 200	hold	add 2000 eliminate 20	hold
3	3633	**without 50** **20, 100, 200, 500, 1000 (5)**	**full series** **20, 50, 100, 200, 500, 1000 (6)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**
		add 200 eliminate 50	re-add 50	add 2000 eliminate 20	hold
4	3333	**without 50** **20, 100, 200, 500, 1000 (5)**	**without 50** **20, 100, 200, 500, 1000 (5)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**
		add 200 eliminate 50	hold	add 50, 2000 replace 20-baht banknote by coin	hold
5	1633	**without 50, 200** **20, 100, 500, 1000 (4)**	**full series** **20, 50, 100, 200, 500, 1000 (6)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**
		eliminate 50	add 50, 200	add 2000 replace 20-baht banknote by coin	hold
6	4413	**without 200** **20, 50, 100, 500, 1000 (5)**	**without 200** **20, 50, 100, 500, 1000 (5)**	**without 200** **50, 100, 500, 1000, 2000 (5)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**
		hold	hold	add 2000 replace 20-baht banknote by coin	add 200
7	1333	**without 50, 200** **20, 100, 500, 1000 (4)**	**without 50** **20, 100, 200, 500, 1000 (5)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**
		eliminate 50	add 200	add 50, 2000 replace 20-baht banknote by coin	hold
8	1323	**without 50, 200** **20, 100, 500, 1000 (4)**	**without 50** **20, 100, 200, 500, 1000 (5)**	**without 500** **50, 100, 200, 1000, 2000 (5)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**
		eliminate 50	add 200	add 50, 2000 replace 20-baht banknote by coin eliminate 500	add 500
9	1413	**without 50, 200** **20, 100, 500, 1000 (4)**	**without 200** **20, 50, 100, 500, 1000 (5)**	**without 200** **50, 100, 500, 1000, 2000 (5)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**
		eliminate 50	re-add 50	add 2000 replace 20-baht banknote by coin	add 200
10	1133	**without 50, 200** **20, 100, 500, 1000 (4)**	**without 50, 200** **20, 100, 500, 1000 (4)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**
		eliminate 50	hold	add 50, 200, 2000 replace 20-baht banknote by coin	hold
11	1311	**without 50, 200** **20, 100, 500, 1000 (4)**	**without 50** **20, 100, 200, 500, 1000 (5)**	**without 200** **50, 100, 500, 1000, 2000 (5)**	**without 200** **50, 100, 500, 1000, 2000 (5)**
		eliminate 50	add 200	add 50, 2000 replace 20-baht banknote by coin eliminate 200	hold
12	1411	**without 50, 200** **20, 100, 500, 1000 (4)**	**without 200** **20, 50, 100, 500, 1000 (5)**	**without 200** **50, 100, 500, 1000, 2000 (5)**	**without 200** **50, 100, 500, 1000, 2000 (5)**
		eliminate 50	re-add 50	add 2000 replace 20-baht banknote by coin	hold
13	1113	**without 50, 200** **20, 100, 500, 1000 (4)**	**without 50, 200** **20, 100, 500, 1000 (4)**	**without 200** **50, 100, 500, 1000, 2000 (5)**	**full series** **50, 100, 200, 500, 1000, 2000 (6)**
		eliminate 50	hold	add 50, 2000 replace 20-baht banknote by coin	add 200
14	1111	**without 50, 200** **20, 100, 500, 1000 (4)**	**without 50, 200** **20, 100, 500, 1000 (4)**	**without 200** **50, 100, 500, 1000, 2000 (5)**	**without 200** **50, 100, 500, 1000, 2000 (5)**
		eliminate 50	hold	add 50, 2000 replace 20-baht banknote by coin	hold

Note: The figures in parentheses denote the number of banknote denominations.

Table 5.18 Performances of Optimal Structures according to Dynamic Analysis

Structure		Production Amount (Millions of Notes)			Total Cost (Millions of Baht)			Cash Payment Efficiency: Average No. of Minimum Tokens		Cash Payment Efficiency: Average No. of Efficient Schemes		Average No. of Banknote Denominations (Period#1/2/3/4)	Average Space of Currency Denominations Including Coins (Period#1/2/3/4)
Scenario	**Code**	**Value**	**Diff**	**%Diff**	**Value**	**Diff**	**%Diff**	**Value**	**%Diff**	**Value**	**%Diff**		
Benchmark	4411	76479	0	0.0%	118519	0	0.0%	5.23	0.0%	1.93	0.0%	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
Optimal 1 (efficiency)	6633	70432	-6048	-7.9%	129828	11309	9.5%	4.92	-5.9%	2.57	32.9%	6.0 (6,6,6,6)	2.14 (2.14,2.14,2.14,2.14)
Optimal 2	3633	70962	-5518	-7.2%	125842	7323	6.2%	5.00	-4.5%	2.41	24.9%	5.8 (5,6,6,6)	2.20 (2.40,2.14,2.14,2.14)
Optimal 3	3333	71570	-4909	-6.4%	122663	4144	3.5%	5.07	-3.1%	2.25	16.8%	5.5 (5,5,6,6)	2.27 (2.40,2.40,2.14,2.14)
Optimal 4	1633	72234	-4245	-5.6%	121589	3070	2.6%	5.09	-2.6%	2.39	24.0%	5.5 (4,6,6,6)	2.28 (2.72,2.14,2.14,2.14)
Optimal 5	4413	75278	-1201	-1.6%	120486	1967	1.7%	5.15	-1.5%	2.10	8.7%	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
Optimal 6	1333	72843	-3637	-4.8%	118410	-109	-0.1%	5.17	-1.2%	2.24	15.9%	5.3 (4,5,6,6)	2.35 (2.72,2.40,2.14,2.14)
Optimal 7	1323	75233	-1246	-1.6%	116726	-1793	-1.5%	5.24	0.2%	2.07	7.2%	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
Optimal 8	1413	75484	-995	-1.3%	116182	-2337	-2.0%	5.25	0.3%	2.07	7.5%	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
Optimal 9	1133	73978	-2501	-3.3%	114936	-3583	-3.0%	5.27	0.7%	2.22	15.0%	5.0 (4,4,6,6)	2.43 (2.72,2.72,2.14,2.14)
Optimal 10	1311	75725	-754	-1.0%	114311	-4208	-3.6%	5.32	1.8%	1.90	-1.5%	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
Optimal 11	1411	76685	206	0.3%	114215	-4304	-3.6%	5.33	1.8%	1.91	-1.2%	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
Optimal 12	1113	75659	-820	-1.1%	112688	-5832	-4.9%	5.34	2.2%	2.05	6.3%	4.8 (4,4,5,6)	2.50 (2.72,2.72,2.40,2.14)
Optimal 13 (cost)	1111	76860	381	0.5%	110721	-7798	-6.6%	5.42	3.6%	1.88	-2.4%	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)

Note: A benchmark is the structure code 4411.

The difference (diff) with a negative sign shows saving from a benchmark.

CHAPTER 6

CONCLUSION

6.1 Summary

This research is on the subject of the optimal currency denomination structure: regarding the case of Thailand, focusing on banknotes. The solution to the optimization problem has been used to verify the current currency denomination structure in Thailand as to whether it is optimal (static analysis). Moreover, it has been further applied to planning for a future currency denomination structure (dynamic analysis). The study concerns all relevant economic agents, which are central banks, commercial banks, and people that use cash. In other words, not only the cost saving target on the supply side but also cash payment efficiency of transactors on the demand side were taken into account. Moreover, economic drivers, such as the macroeconomic condition and consumer preferences, were considered. Consequently, the model seems to be massive, as it is composed of many components, i.e. banknote demand by denomination, cash payment profile, cash payment efficiency, and the projection of relevant economic factors. Altogether, the research question was solved using the simulation-optimization technique based on cost and efficiency perspectives. However, two main components focused on in this study are banknote demand by denomination and the optimization process with simulation.

The characteristic model is applied to the analysis of banknote demand by denomination. The results show that the factors which would significantly explain the banknote demand share are size, average spread, value with the power of ten, number of banknote denominations, average space, lowest banknote denomination, highest banknote denomination, and nominal GDP per capita. The estimated demand function allows for determining the demand for banknote denominations which do not exist at this time. This demand function is therefore applied in this research in order to determine the demand share by denomination in various scenarios with the simulation of denomination structures together with the prediction of banknote characteristics and macroeconomic factors. Regarding the optimization problem, cost is selected to be minimized under cash payment efficiency constraints. In the case that the cash payment efficiency is restricted by the average number of minimum tokens to be not greater than the current situation or the benchmark (in other words, the cash payment efficiency must not be worse off), the selected optimal banknote denomination structure from the static analysis is 20-100-200-500-1000. On the other hand, the solution from the dynamic analysis proposes different optimal structures during twenty-year periods focused on in the study, i.e. 20-100-500-1000 for 2011-2015, 20-100-200-500-1000 for 2015-2020, and 50-100-200-500-1000-2000 for 2021-2030. It was learned from the static analysis that the current structure is not optimal according to the cost and cash payment efficiency perspectives. And the proposed re-structuring plan from the dynamic analysis would improve cost and cash payment efficiency by 0.1% and 1.2%, respectively.

6.2 Research Recommendations

6.2.1 Policy Recommendations

Referring to the research, the dynamically numerical analysis suggests the 20-year forward plan; that is, 50-baht banknotes should be eliminated today followed by introducing 200-baht banknotes in the next five years. After that, 20- baht banknotes should be replaced by 20- baht coins together with introducing 2000-baht banknotes and re-entering 50-baht banknotes in 2021 until the end of the study period (2030). This suggestion is only a guideline for policymakers to plan for the re-structuring of banknote denominations according to the economic optimization perspective. In addition, the methodology applied in this research could be an alternative for Bank of Thailand in ascertaining an optimal currency denomination structure for Thailand when relevant factors are taken into account, for example e-payments, consumer preferences, cash payment profile, etc.

However, because there are many factors or drivers which affect circumstances and consumers' behavior over time, the numerical analysis should be dynamically re-examined in order to make sure that the update situation is determined with some shocks, such as credit cards and financial crises, have been taken into account.

Policymakers can conduct in-depth analyses by using information such as cost and production figures to obtain a more accurate optimal solution regarding the optimal currency denomination structure. For reasons of confidentiality, this research uses some estimated information obtained from authorized institutions, such as cost information. If more detailed information were available, the research can be revised at any time in order to provide a more reliable solution. Nevertheless, the re-structuring plan could be flexible according to how the significance weighed between cost savings on the supply side and cash payment efficiency achieved by all transactors is determined. This is because many optimal structures vary by the level of cost and efficiency, which are a trade-off of each other. The solution therefore depends on policymakers identifying the extent to which each scheme (cost and cash payment efficiency) is emphasized.

6.2.2 Further Study

Many assumptions and limitations were employed in narrowing down the scope of this study. This research focuses on demand analysis with a characteristic model and optimization with the simulation-optimization technique. The other parts, which are the exogenous variable forecast, boundary verification by D-Metric and payment profile, and cash payment efficiency, are roughly estimated in the research. In order to obtain more accurate outcome, all components therefore have to be studied in detail in order to solve this multi-component problem.

First, a cash payment profile should be investigated in order to obtain a realistic distribution, which should not be uniform, as assumed in the research. Regarding the cash payment efficiency determination in this research, each payment amount has its own efficiency value in two aspects; a payment profile is therefore required for averaging the efficiency values. However, a more reliable and accurate average efficiency value can be obtained if a realistic distribution of the cash payment amount is applied. A survey to capture the cash payment profile is therefore targeted for further study.

Second, the denomination boundary should be carefully identified, and should not only refer to the D-Metric model and experts' judgments. In this research, the boundary and the series inside a structure were separately determined. Actually, focus was placed on the latter point and the boundary according to the D-Metric model was roughly estimated to be a guideline together with the judgments of experts. However, the D-Metric model still has some limitations; for instance, it neglects cultural preferences, other methods of payment (debit and credit cards), and average transaction size. Future study therefore should include fine-tuning the boundary. This can be done by using the analyses of either historical data or pooled data across countries via the econometric model with appropriate explanatory variables, such as e-payment.

Third, the projections of macroeconomic variables, for example, GDP, population, and total banknote supply, should be re-examined by setting up an appropriate model in both functional form and estimation method, including a set of explanatory variables. For example, in order to predict total banknote supply, credit card usage, which can be a proxy of e-payment, would be taken into the model. Another example is the projection of the GDP, which could be achieved via a macroeconomic model.

Fourth, in the cash payment efficiency analysis, "wallet constraint" should be imposed in an algorithm for verifying cash payment efficiency so that it corresponds to a more realistic situation. An important limitation used in cash payment efficiency is the assumption that people have all denominations in their wallet with unlimited amounts. In other words, they have no wallet constraints and are able to make a payment with minimum tokens, which is the efficient condition according to the principle of least effort. Wallet constraint can be taken into account and the algorithm has to be changed. In addition, the simulation of cash in wallets maybe arranged to mimic the real situation of the users.

Finally, cash payment efficiency should be valued and transformed to money by the valuation technique. This would allow cost and efficiency to be offset and represented by net economic loss or gain. Cash payment efficiency may therefore be applied to be an objective function, which includes cost and efficiency together. In this way, an optimal solution would be clearly obtained with one solution. Moreover, money savings would be expected to be higher according to the results of this study because some additional gains from the cash payment efficiency perspective are included.

6.3 Application to Other Areas

According to the research, which attempts to solve problems with optimal denomination structure, the methodology may be able to be applied to other research questions from other fields besides the monetary area, for example, wage and labor, health and medicine, education, transportation, politics, telecommunications, and energy products. Applied research questions should be topics concerning the allocation of public goods in a structure according to which consumers cannot make a decision and are forced to accept that structure, such as the denomination structure. Moreover the sub-products or goods in a structure should be able to be nearly perfectly substituted for each other, such as various currency denominations. Examples of possible application are the supply for mobile phone systems (GSM, DTAC. TRUEMOVE), broadband Internet (TRUE, TOT, 3BB), car fuel (Benzene, Gasohol, LPG, NGV), bus (BMTA, Micro bus), and political parties.

Referring to the research framework shown in Figure 3.1, the methodology structure for general application is summarized in Figure 6.1.

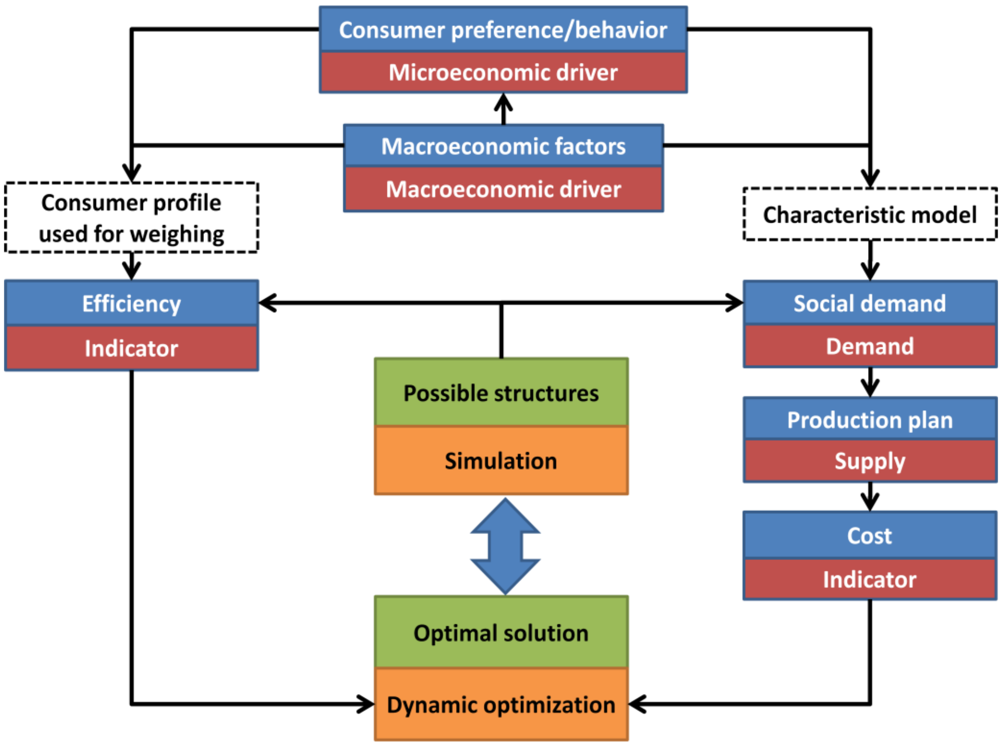

Figure 6.1 The Methodology for General Application

The diagram is composed of four main components, i.e. supply, demand, indicators, and drivers. The details are shown as follows:

6.3.1 Demand

The demand is the component that represents the consumers' preferences and their needs. According to our study, the demand function is estimated by a characteristic model using historical data as a reflection of social demand.

6.3.2 Supply

With social demand, an equilibrium condition might be assumed to imply supply and be able to estimate a production plan for goods.

6.3.3 Indicators

The indicators are used for determining the performance of a given structure, e.g. cost and efficiency. They can be used to be an objective function to be optimized or a constraint in an optimization problem. In our study, cost and cash payment efficiency are the indicators used for being an objective function to be optimized and a constraint, respectively.

6.3.4 Drivers

The drivers are the exogenous factors which finally affect the indicators. The changes in indicators imply that the optimal structure should be reviewed and revised. From the diagram, there are both microeconomic and macroeconomic drivers in which microeconomic drivers might be influenced by macroeconomic drivers.

With the four components discussed, the scenarios are dynamically simulated. The optimization problem will be solved by comparison among scenarios. The solution is then the one which makes the objective indicator optimized under the imposed constraints.

In summary, this research can be applied to other fields. With the proposed methodology, it can be seen that macroeconomic and microeconomic perspectives are incorporated in both demand and supply sides to be able to access all economic agents.

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APPENDICES

APPENDIX A

Bachet's Problem[29]

Bachet's problem of the weights. What is the least number of weights which will weigh any integral number of pounds up to 40 (a) when weights may be put into one pan only and (b) when weights may be put into either pan?

The second problem is the more interesting. We can dispose of the first by proving

Theorem 1. Weights 1, 2, 4, …, 2^{n-1} will weigh any integral weight up to 2^n-1; and no other set of so few as n weights is equally effective (i.e. will weight so long an unbroken sequence of weights from 1)

Any positive integer up to 2^n-1 inclusive can be expressed uniquely as a binary decimal of n figures, i.e. as a sum

$$\sum_0^{n-1} a_s 2^s ,$$

where every a, is 0 or 1. Hence our weights will do what is wanted, and 'without waste' (no two arrangements of them producing the same result). Since there is no waste, no other selection of weights can weigh a longer sequence.

Finally, one weight must be 1 (to weigh 1); one must be 2 (to weigh 2); one must be 4 (to weigh 4); and so on. Hence 1, 2, 4, …, 2^{n-1} is the only system of weights which will do what is wanted.

It is to be observed that Bachet's number 40, not being of the form 2^n-1, is not chosen appropriately for this problem. The weights 1, 2, 4, 8, 16, 32 will weigh up to

[29] Source: An Introduction to the Theory of Numbers by Hardy, Wright

63, and no combination of 5 weights will weigh beyond 32. But the solution for 40 is not unique; the weights 1, 2, 4, 8, 9, 16 will also weigh any weight up to 40.
Passing to the second problem, we prove

Theorem 2. Weights 1, 3, 9, …, 3^{n-1} will weigh any weight up to $(3^n-1)/2$, when weights may be placed in either pan; and no other set of so few as n weights is equally effective.

(1) Any positive integer up to 3^n-1 inclusive can be expressed uniquely by 2 digits in the ternary scale, i.e. as a sum

$$\sum_{0}^{n-1} a_s 3^s ,$$

where every a, is 0,1, or 2. Subtracting

$$1 + 3 + 3^2 + \ldots + 3^{n-1} = (3^n-1)/2,$$

We see that every positive or negative integer between $-(3^n-1)/2$ and $(3^n-1)/2$ inclusive can be expressed uniquely in the form

$$\sum_{0}^{n-1} b_s 3^s ,$$

where every b_s, is -1, 0,or 1. Hence our weights, placed in either pan, will weigh any weight between these limits. Since there is no waste, no other combination of n weights can weigh a longer sequence.

(2) The proof that no other combination will weigh so long a sequence is a little more troublesorne. It is plain, since there must be no waste, that the weights must all differ. We suppose that they are

$$w_1 < w_2 < \ldots < w_,.$$

The two largest weighable weights are plainly

$$w = w_1 + w_2 + \ldots + w_n, \qquad w_1 = w_2 + \ldots + w_n$$

Since $w_1 = w - 1$, w_1 must be 1

The next weighable weight is

$$-w_1 + w_2 + w_3 + \ldots + w_n = w - 2$$

And the next must be

$$w_1 + w_3 + w_4 + \ldots + w_n$$

Hence $w_1 + w_3 + \ldots + w_n = w - 3$.

Suppose now that we have proved that

$$w_1 = 1, w_2 = 3, \ldots > w_s = 3^{s-1}.$$

If we can prove that $w_{s+1} = 3^s$, the conclusion will follow by induction.

The largest weighable weight w is

$$w = \sum_{1}^{s} w_t + \sum_{s+1}^{n} w_t$$

Leaving the weights $w_{s+1}, \ldots , w_n$ undisturbed, and removing some of the other weights, or transferring them to the other pan, we can weigh every weigh down to

$$-\sum_{1}^{s} w_t + \sum_{s+1}^{n} w_t = w - (3^s\text{-}1),$$

but none below. The next weight less than this is $w - 3^s$, and this must be

$$w_1 + w_2 + \ldots + w_s + w_{s+2} + w_{s+3} + \ldots + w_n$$

Hence

$$w_{s+1} = 2(w_1 + w_2 + \ldots + w_s) + 1 = 3^s$$

The conclusion required.

Bachet's problem corresponds to the case n = 4

APPENDIX B

Cramer's Approach[30]

Cramer (1983) formulated efficient payments in mathematical terms as the solution to an optimizing problem. Consider *A* to be the amount to be paid, and *n(A)* the combination of the different notes and coins used in the cash payment. If the different denominations in an arbitrary currency range are numbered as $d = 1, \ldots, D$, then *n(A,d)* denotes the number of tokens of denomination d used for paying amount A. A positive *n(A,d)* refers to use as a payment, while a negative *n(A,d)* means that the n tokens of denomination d are given as change. We denote the value of denomination d by *v(d)*. Efficient payments *n(A,d)* are then the solutions to the following problem:

Minimize

$$n(A) = \sum_{d} |n(A,d)|$$

s.t.

$$\sum_{d} n(A,d) v(d) = A$$

Given the values for *v*, this problem is solved as followed. We take a range of amounts that are of interest. The goal of the algorithm is to cover each amount in this list with an efficient combination of tokens. The steps of the algorithm are as follows:

1) The algorithm starts by covering all amounts in the list that can be paid by only one token.

2) Next, all amounts that can be paid with 2 tokens, either given by the consumer as payment or by the retailer as change, are computed. If in this step we find an amount that was already covered with only one token in the previous step, we

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[30] Source: Empirical Studies on Cash Payments by Jeanine Kippers et al.

do not add this pair of two tokens to the list, since two tokens is not efficient for this amount.

3) To the pairs that were found efficient in the previous step, we add each token once, both with positive and negative sign. For a given pair, these results in $2D$ extra potential combinations with an additional token each. Adding a token with a positive sign to a combination, which has this same token with a negative sign, would yield a combination with less tokens and is therefore ignored. Also, we have the restriction that the highest token has a positive sign. With these combinations, we cover all resulting amounts, provided they were not already covered by fewer tokens.

4) We repeat step 3) until all amounts on the list are covered.

This algorithm results in a number of efficient combinations for each possible amount in the range specified. Many amounts can be paid efficiently with more than one combination.

APPENDIX C

History of Thai Currency Denomination Structure[31]

The currency was originally known as the *tical*; this name was used in the English language text on banknotes until 1925. However, the name baht was established as the Thai name by the 19th century. Both tical and baht were originally units of weight and coins were issued in both silver and gold denominated by their weight in baht and its fractions and multiples. Until 1897, the baht was subdivided into 8 *fuang* (เฟือง), each of 8 *ath* (อัฐ). Other denominations in use were:

Denomination	Value	
bia	1⁄6400 Baht	cowrie; a very small amount of money;
solot	1⁄128 Baht	
att or ath	1⁄64 Baht	
sio or py	1⁄32 Baht	a quarter (feuang)
sik	1⁄16 Baht	a section; a half (feuang)
feuang	1⁄8 Baht	
salung	1⁄4 Baht	a quarter (baht)
mayon	1⁄2 Baht	
baht	1 Baht	1 tical, from Portuguese, from Malay *tikal*
tamleung	4 baht	a gourd; weight of silver equal to four baht, or ~60 grams
chang	20 tamleung or 80 baht	a catty ~1200 gram weight of silver; as a metric unit of weight, chang luang ชั่งหลวง = 600 grams
hap	80 chang or 6400 baht	~96 kg of silver, roughly equivalent to the monetary talent; hap luang หาบหลวง = 60 kg

[31] from wikipedia

The decimal system devised by Prince Mahisorn, in which 1 baht = 100 satang, was introduced by king Chulalongkorn in 1897. However, coins denominated in the old units were issued until 1910. One hangover from the pre-decimalization system: the 25 satang (¼ baht) is still colloquially called a *salueng* or *salung* (สลึง). It is occasionally used for amounts not exceeding 10 salueng or 2.50 baht. A 25-satang coin is also sometimes called *salueng coin.* Until November 27, 1902, the tical was fixed on a purely silver basis, with 15 grams of silver to the baht. This caused the value of the currency to vary relative to currencies on a gold standard. In 1857, the values of certain foreign silver coins were fixed in law, with the 1 baht = 0.6 Straits dollar and 5 baht = 7 Indian rupees. Before 1880 the exchange rate was fixed at eight baht per pound sterling, falling to ten to the pound during the 1880s. In 1902, the government began to increase the value of the baht by following all increases in the value of silver against gold but not reducing it when the silver price fell. Beginning at 21.75 baht = 1 British pound, the currency rose in value until, in 1908, a fixed peg to the British pound was established of 13 baht = 1 pound. This was revised to 12 baht in 1919 and then, after a period of instability, to 11 baht in 1923. During the Second World War, the baht was fixed at a value of 1 Japanese yen. From 1956 until 1973, the baht was pegged to the U.S. dollar at an exchange rate of 20.8 baht = 1 dollar and at 20 baht = 1 dollar until 1978. A strengthening US economy caused Thailand to re-peg its currency at 25 to the dollar from 1984 until July 2, 1997, when the country was stung by the Asian financial crisis. The baht was floated and halved in value, reaching its lowest rate of 56 to the dollar in January 1998. It has since risen to about 32 per dollar.

C.1 Coins

Before 1860, Thailand did not produce coins using modern methods. Instead, a so-called "bullet" coinage was used, consisting of bars of metal, thicker in the middle, bent round to form a complete circle on which identifying marks were stamped. Denominations issued included $^1⁄_{128}$, $^1⁄_{64}$, $^1⁄_{32}$, $^1⁄_{16}$, ⅛, ½, 1, 1½, 2, 2½, 4, 4½, 8, 10, 20, 40, and 80 baht in silver and $^1⁄_{32}$, $^1⁄_{16}$, ⅛, ½, 1, 1½, 2, and 4 baht in gold. Between 1858 and 1860, foreign trade coins were also stamped by the government for use in

Thailand. In 1860, modern style coins were introduced. These were silver 1 sik, 1 fuang, 1 and 2 salung, 1, 2 and 4 baht, with the baht weighing 15.244 grams and the others weight related. Tin 1 solot and 1 att followed in 1862, with gold 2½, 4 and 8 baht introduced in 1863 and copper 2 and 4 att in 1865. Copper replaced tin in the 1 solot and 1 att in 1874, with copper 4 att introduced in 1876. The last gold coins were struck in 1895. In 1897, the first coins denominated in satang were introduced, cupro-nickel 2½, 5, 10 and 20 satang. However, 1 solot, 1 and 2 att coins were struck until 1905 and 1 fuang coins were struck until 1910. In 1908, holed 1, 5 and 10 satang coins were introduced, with the 1 satang in bronze and the 5 and 10 satang in nickel. The 1 and 2 salung were replaced by 25 and 50 satang coins in 1915. In 1937, holed, bronze ½ satang were issued. In 1941, a series of silver coins was introduced in denominations of 5, 10 and 20 satang, due to a shortage of nickel caused by WWII. The next year, tin coins were introduced for 1, 5 and 10 satang, followed by 20 satang in 1945 and 25 and 50 satang in 1946. In 1950, aluminium-bronze 5, 10, 25 and 50 satang were introduced whilst, in 1957, bronze 5 and 10 satang were issued, along with 1 baht coins struck in an unusual alloy of copper, nickel, silver and zinc. It should be notes that several Thai coins were issued for many years without changing the date. These include the tin 1942 1 satang and the 1950 5 and 10 satang, struck until 1973, the tin 1946 25 satang struck until 1964, the tin 50 satang struck until 1957, and the aluminium bronze 1957 5, 10, 25 and 50 satang struck until the 1970s. Cupro-nickel 1 baht coins were introduced in 1962 and struck without date change until 1982. In 1972, cupro-nickel 5 baht coins were introduced, switching to cupro-nickel-clad copper in 1977. Between 1986 and 1988, a new coinage was introduced, consisting of aluminium 1, 5 and 10 satang, aluminium-bronze 25 and 50 satang, cupro-nickel 1 baht, cupro-nickel-clad-copper 5 baht and bimetallic 10 baht. Cupro-nickel-clad-steel 2 baht were introduced in 2005.

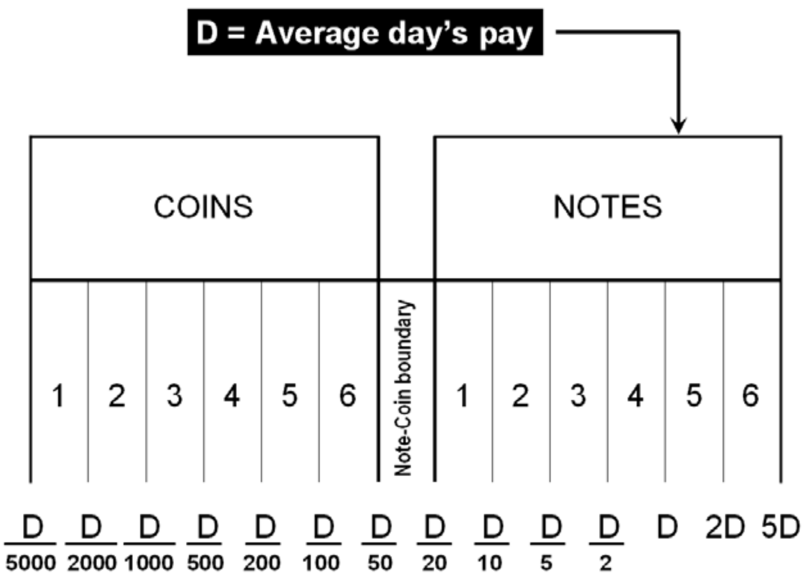
C.2 Banknotes

In 1851, the government issued notes for ⅛, ¼, ⅜, ½ and 1 tical, followed by 3, 4, 6 and 10 tamlung in 1853. After 1857, notes for 20 and 40 ticals were issued, also bearing their values in Straits dollars and Indian rupees. Undated notes were also issued before 1868 for 5, 7, 8, 12 and 15 tamlung, and 1 chang. 1 att notes were issued in 1874. In 1892, the Treasury issued notes for 1, 5, 10, 40, 80, 100, 400 and 800 ticals, called baht in the Thai text. On September 19, 1902, the government introduced notes for 5, 10, 20, 100 and 1000 ticals, with 1 and 50 ticals notes following in 1918. In 1925, notes were issued with the denomination baht used in the English text, in denominations of 1, 5, 10, 20, 100 and 1000 baht. In 1942, the Bank of Thailand as founded and took over responsibility for the issuance of paper money. 50 baht notes were briefly reintroduced in 1945, with 50 satang notes issued in 1946. The 1 baht note was replaced by a coin in 1957 and the 5 baht was replaced in 1972. 50 baht note were again reintroduced in 1985, with the 10 baht note replaced by a coin in 1988.

APPENDIX D

D-Metric Model[32]

"D-Metric model," was developed in 1981 by L C Payne and H M Morgan. It is based on the empirical relationship between the average day's net pay and a currency's denomination structure. Since it was developed, a number of countries have used it to adjust the denomination structure of their currency.



Analysis of the denomination structures of a wide range of countries and the average wage prevailing in these countries, reveals a remarkably consistent pattern between the average day’s pay (D) and denomination structure. From a diagram, it was found that the top banknote denomination in most countries is around 5D, the transition between coins and banknotes is between D/50 and D/20 and the lowest useful coin is around D/5000.

[32] Source: UK Currency Needs in the 1980s by L C Payne and H M Morgan & Currency trends and development by John Barry

APPENDIX E

Empirical Results according to Characteristic Model

Initial Model:

FGNLS regression

	Equation	Obs	Parms	RMSE	R-sq	Constant
1	s_v10	30	13	.0203005	0.4606*	(none)
2	s_v20	30	13	.0191494	0.5315*	(none)
3	s_v100	30	13	.0694567	0.9039*	(none)
4	s_v500	30	13	.0701549	0.9765*	(none)
5	s_v1000	30	13	.0425509	0.9918*	(none)

* Uncentered R-sq

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
/c2	.0000164	.0001886	0.09	0.931	-.0003532	.000386
/c3	-.9672066	.7534502	-1.28	0.199	-2.443942	.5095286
/c4	.5587151	.2923585	1.91	0.056	-.014297	1.131727
/c5	.0000151	.0000136	1.11	0.268	-.0000116	.0000418
/c6	-5.715187	1.561511	-3.66	0.000	-8.775693	-2.654681
/c7	1.488579	.5973763	2.49	0.013	.3177425	2.659415
/c8	-.0111173	.0089023	-1.25	0.212	-.0285654	.0063308
/c9	-3.065297	3.822314	-0.80	0.423	-10.5569	4.426301
/c10	.4960096	.1684039	2.95	0.003	.1659441	.8260751
/c11	-.4084837	9.342843	-0.04	0.965	-18.72012	17.90315
/c12	.5189379	2.906404	0.18	0.858	-5.177509	6.215385
/c13	6.611139	.	.	.	.	.
/c1	.0548027	.0088841	6.17	0.000	.0373901	.0722152

Improved Model:

FGNLS regression

	Equation	Obs	Parms	RMSE	R-sq	Constant
1	s_v10	30	9	.0089616	0.8949*	(none)
2	s_v20	30	9	.0080119	0.9180*	(none)
3	s_v100	30	9	.0679492	0.9080*	(none)
4	s_v500	30	9	.1426201	0.9030*	(none)
5	s_v1000	30	9	.1351696	0.9170*	(none)

* Uncentered R-sq

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
/c2	1.54e-06	4.29e-07	3.58	0.000	6.96e-07	2.38e-06
/c3	.1318171	.0114114	11.55	0.000	.1094512	.154183
/c4	.0089982	.0034286	2.62	0.009	.0022784	.0157181
/c5	-.000662	.0000621	-10.66	0.000	-.0007837	-.0005403
/c6	.2018817	.0257777	7.83	0.000	.1513583	.2524052
/c7	-.0144747	.0063866	-2.27	0.023	-.0269921	-.0019573
/c8	.4573497	.0805329	5.68	0.000	.2995082	.6151912
/c9	.0754318	.0225369	3.35	0.001	.0312603	.1196032
/c1	.9999996	.	.	.	.	.

APPENDIX F

D-Metric Model Applied for Thailand

	Coin							Banknote						
	D/5000	D/2000	D/1000	D/500	D/200	D/100	D/50	D/20	D/10	D/5	D/2	D	2D	5D
		0.25	0.50	1	2	5	10	20	50	100	200	500	1000	
2011	0.09	0.22	0.43	0.86	2.16	4.32	8.64	22	43	86	216	432	864	2159
2012	0.09	0.23	0.46	0.92	2.30	4.59	9.19	23	46	92	230	459	919	2297
2013	0.10	0.24	0.49	0.98	2.44	4.89	9.77	24	49	98	244	489	977	2443
2014	0.10	0.26	0.52	1.04	2.60	5.20	10.39	26	52	104	260	520	1039	2598
2015	0.11	0.28	0.55	1.10	2.76	5.52	11.05	28	55	110	276	552	1105	2762
2016	0.12	0.29	0.59	1.18	2.94	5.89	11.77	29	59	118	294	589	1177	2943
2017	0.13	0.31	0.63	1.25	3.13	6.27	12.54	31	63	125	313	627	1254	3135
2018	0.13	0.33	0.67	1.34	3.34	6.68	13.36	33	67	134	334	668	1336	3340
2019	0.14	0.36	0.71	1.42	3.56	7.12	14.24	36	71	142	356	712	1424	3560
2020	0.15	0.38	0.76	1.52	3.79	7.59	15.18	38	76	152	379	759	1518	3794
		0.50	1	2	5	10	20	50	100	200	500	1000	2000	
2021	0.16	0.40	0.81	1.62	4.05	8.10	16.20	40	81	162	405	810	1620	4049
2022	0.17	0.43	0.86	1.73	4.32	8.64	17.28	43	86	173	432	864	1728	4321
2023	0.18	0.46	0.92	1.84	4.61	9.22	18.45	46	92	184	461	922	1845	4612
2024	0.20	0.49	0.98	1.97	4.92	9.85	19.69	49	98	197	492	985	1969	4923
2025	0.21	0.53	1.05	2.10	5.25	10.51	21.02	53	105	210	525	1051	2102	5254
2026	0.22	0.56	1.12	2.25	5.62	11.23	22.47	56	112	225	562	1123	2247	5617
2027	0.24	0.60	1.20	2.40	6.00	12.01	24.02	60	120	240	600	1201	2402	6005
2028	0.26	0.64	1.28	2.57	6.42	12.84	25.68	64	128	257	642	1284	2568	6419
2029	0.27	0.69	1.37	2.74	6.86	13.72	27.45	69	137	274	686	1372	2745	6862
2030	0.29	0.73	1.47	2.93	7.34	14.67	29.34	73	147	293	734	1467	2934	7336

Note: D values have been estimated by nominal GDP per capita per day**.**

APPENDIX G

The Summary of Dynamic Analysis Simulation

Code	Total Cost 2011-2030 (Millions of Baht)	Cash Payment Efficiency		Average Number of Denominations (Period# 1/2/3/4)	Average Spacing Factor (Period# 1/2/3/4)
		Average No. of Minimum Tokens	Average No. of Efficient Schemes		
1111	110721	5.42	1.88	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
1112	111033	5.42	1.88	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
1113	112688	5.34	2.05	4.8 (4,4,5,6)	2.50 (2.72,2.72,2.40,2.14)
1121	111377	5.42	1.88	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
1122	111507	5.42	1.88	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
1123	113252	5.34	2.05	4.8 (4,4,5,6)	2.50 (2.72,2.72,2.40,2.14)
1131	113061	5.34	2.05	4.8 (4,4,6,5)	2.50 (2.72,2.72,2.14,2.40)
1132	113282	5.34	2.05	4.8 (4,4,6,5)	2.50 (2.72,2.72,2.14,2.40)
1133	114936	5.27	2.22	5.0 (4,4,6,6)	2.43 (2.72,2.72,2.14,2.14)
1211	111515	5.42	1.87	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
1212	111828	5.42	1.87	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
1213	113482	5.35	2.03	4.8 (4,4,5,6)	2.50 (2.72,2.72,2.40,2.14)
1221	111938	5.42	1.87	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
1222	112068	5.42	1.87	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
1223	113814	5.34	2.03	4.8 (4,4,5,6)	2.50 (2.72,2.72,2.40,2.14)
1231	113739	5.35	2.03	4.8 (4,4,6,5)	2.50 (2.72,2.72,2.14,2.40)
1232	113960	5.34	2.03	4.8 (4,4,6,5)	2.50 (2.72,2.72,2.14,2.40)
1233	115614	5.27	2.20	5.0 (4,4,6,6)	2.43 (2.72,2.72,2.14,2.14)
1311	114311	5.32	1.90	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1312	114624	5.32	1.90	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1313	116278	5.25	2.07	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
1321	114851	5.32	1.90	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1322	114980	5.32	1.90	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1323	116726	5.24	2.07	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
1331	116535	5.25	2.07	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
1332	116756	5.24	2.07	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
1333	118410	5.17	2.24	5.3 (4,5,6,6)	2.35 (2.72,2.40,2.14,2.14)
1411	114215	5.33	1.91	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1412	114527	5.32	1.91	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1413	116182	5.25	2.07	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
1421	114871	5.32	1.91	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1422	115001	5.32	1.91	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1423	116746	5.25	2.07	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
1431	116555	5.25	2.07	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
1432	116776	5.25	2.07	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
1433	118431	5.17	2.24	5.3 (4,5,6,6)	2.35 (2.72,2.40,2.14,2.14)
1511	114780	5.35	2.04	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1512	115092	5.35	2.04	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1513	116746	5.27	2.20	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
1521	115202	5.35	2.04	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1522	115332	5.34	2.04	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
1523	117078	5.27	2.20	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
1531	117003	5.27	2.20	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
1532	117224	5.27	2.20	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
1533	118879	5.19	2.37	5.3 (4,5,6,6)	2.35 (2.72,2.40,2.14,2.14)
1611	117490	5.25	2.06	5.0 (4,6,5,5)	2.41 (2.72,2.14,2.40,2.40)
1612	117802	5.25	2.06	5.0 (4,6,5,5)	2.41 (2.72,2.14,2.40,2.40)
1613	119457	5.17	2.22	5.3 (4,6,5,6)	2.35 (2.72,2.14,2.40,2.14)
1621	118029	5.25	2.06	5.0 (4,6,5,5)	2.41 (2.72,2.14,2.40,2.40)
1622	118159	5.24	2.06	5.0 (4,6,5,5)	2.41 (2.72,2.14,2.40,2.40)
1623	119905	5.17	2.22	5.3 (4,6,5,6)	2.35 (2.72,2.14,2.40,2.14)
1631	119713	5.17	2.22	5.3 (4,6,6,5)	2.35 (2.72,2.14,2.14,2.40)
1632	119934	5.17	2.22	5.3 (4,6,6,5)	2.35 (2.72,2.14,2.14,2.40)
1633	121589	5.09	2.39	5.5 (4,6,6,6)	2.28 (2.72,2.14,2.14,2.14)

The Summary of Dynamic Analysis Simulation (Continued)

Code	Total Cost 2011-2030 (Millions of Baht)	Cash Payment Efficiency		Average Number of Denominations (Period# 1/2/3/4)	Average Spacing Factor (Period# 1/2/3/4)
		Average No. of Minimum Tokens	Average No. of Efficient Schemes		
2111	111706	5.42	1.87	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
2112	112019	5.42	1.87	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
2113	113673	5.35	2.03	4.8 (4,4,5,6)	2.50 (2.72,2.72,2.40,2.14)
2121	112362	5.42	1.87	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
2122	112492	5.42	1.87	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
2123	114238	5.34	2.03	4.8 (4,4,5,6)	2.50 (2.72,2.72,2.40,2.14)
2131	114046	5.35	2.03	4.8 (4,4,6,5)	2.50 (2.72,2.72,2.14,2.40)
2132	114267	5.34	2.03	4.8 (4,4,6,5)	2.50 (2.72,2.72,2.14,2.40)
2133	115922	5.27	2.20	5.0 (4,4,6,6)	2.43 (2.72,2.72,2.14,2.14)
2211	112203	5.43	1.85	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
2212	112515	5.42	1.85	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
2213	114170	5.35	2.01	4.8 (4,4,5,6)	2.50 (2.72,2.72,2.40,2.14)
2221	112626	5.42	1.85	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
2222	112755	5.42	1.85	4.5 (4,4,5,5)	2.56 (2.72,2.72,2.40,2.40)
2223	114501	5.35	2.01	4.8 (4,4,5,6)	2.50 (2.72,2.72,2.40,2.14)
2231	114426	5.35	2.01	4.8 (4,4,6,5)	2.50 (2.72,2.72,2.14,2.40)
2232	114648	5.35	2.01	4.8 (4,4,6,5)	2.50 (2.72,2.72,2.14,2.40)
2233	116302	5.27	2.18	5.0 (4,4,6,6)	2.43 (2.72,2.72,2.14,2.14)
2311	115148	5.33	1.88	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2312	115460	5.32	1.88	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2313	117114	5.25	2.05	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
2321	115687	5.32	1.88	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2322	115817	5.32	1.88	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2323	117562	5.25	2.05	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
2331	117371	5.25	2.05	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
2332	117592	5.25	2.05	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
2333	119247	5.17	2.22	5.3 (4,5,6,6)	2.35 (2.72,2.40,2.14,2.14)
2411	115200	5.33	1.89	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2412	115513	5.33	1.89	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2413	117167	5.25	2.06	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
2421	115856	5.33	1.89	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2422	115986	5.32	1.89	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2423	117732	5.25	2.06	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
2431	117540	5.25	2.06	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
2432	117761	5.25	2.06	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
2433	119416	5.17	2.22	5.3 (4,5,6,6)	2.35 (2.72,2.40,2.14,2.14)
2511	115467	5.35	2.02	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2512	115780	5.35	2.02	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2513	117434	5.27	2.19	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
2521	115890	5.35	2.02	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2522	116020	5.35	2.02	4.8 (4,5,5,5)	2.48 (2.72,2.40,2.40,2.40)
2523	117765	5.27	2.19	5.0 (4,5,5,6)	2.41 (2.72,2.40,2.40,2.14)
2531	117691	5.27	2.19	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
2532	117912	5.27	2.19	5.0 (4,5,6,5)	2.41 (2.72,2.40,2.14,2.40)
2533	119566	5.20	2.35	5.3 (4,5,6,6)	2.35 (2.72,2.40,2.14,2.14)
2611	118326	5.25	2.04	5.0 (4,6,5,5)	2.41 (2.72,2.14,2.40,2.40)
2612	118639	5.25	2.04	5.0 (4,6,5,5)	2.41 (2.72,2.14,2.40,2.40)
2613	120293	5.17	2.21	5.3 (4,6,5,6)	2.35 (2.72,2.14,2.40,2.14)
2621	118866	5.25	2.04	5.0 (4,6,5,5)	2.41 (2.72,2.14,2.40,2.40)
2622	118995	5.25	2.04	5.0 (4,6,5,5)	2.41 (2.72,2.14,2.40,2.40)
2623	120741	5.17	2.21	5.3 (4,6,5,6)	2.35 (2.72,2.14,2.40,2.14)
2631	120550	5.17	2.21	5.3 (4,6,6,5)	2.35 (2.72,2.14,2.14,2.40)
2632	120771	5.17	2.21	5.3 (4,6,6,5)	2.35 (2.72,2.14,2.14,2.40)
2633	122425	5.10	2.37	5.5 (4,6,6,6)	2.28 (2.72,2.14,2.14,2.14)

The Summary of Dynamic Analysis Simulation (Continued)

Code	Total Cost 2011-2030 (Millions of Baht)	Cash Payment Efficiency		Average Number of Denominations (Period# 1/2/3/4)	Average Spacing Factor (Period# 1/2/3/4)
		Average No. of Minimum Tokens	Average No. of Efficient Schemes		
3111	115123	5.32	1.90	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
3112	115435	5.32	1.90	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
3113	117090	5.25	2.07	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
3121	115779	5.32	1.90	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
3122	115909	5.32	1.90	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
3123	117654	5.24	2.07	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
3131	117463	5.25	2.07	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
3132	117684	5.24	2.07	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
3133	119338	5.17	2.24	5.3 (5,4,6,6)	2.35 (2.40,2.72,2.14,2.14)
3211	115768	5.33	1.88	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
3212	116081	5.32	1.88	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
3213	117735	5.25	2.05	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
3221	116191	5.32	1.88	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
3222	116321	5.32	1.88	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
3223	118067	5.25	2.05	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
3231	117992	5.25	2.05	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
3232	118213	5.25	2.05	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
3233	119867	5.17	2.22	5.3 (5,4,6,6)	2.35 (2.40,2.72,2.14,2.14)
3311	118564	5.23	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3312	118877	5.22	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3313	120531	5.15	2.09	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
3321	119104	5.22	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3322	119233	5.22	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3323	120979	5.15	2.09	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
3331	120788	5.15	2.09	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
3332	121009	5.15	2.09	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
3333	122663	5.07	2.25	5.5 (5,5,6,6)	2.27 (2.40,2.40,2.14,2.14)
3411	118617	5.23	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3412	118929	5.23	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3413	120584	5.15	2.09	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
3421	119273	5.23	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3422	119403	5.22	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3423	121148	5.15	2.09	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
3431	120957	5.15	2.09	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
3432	121178	5.15	2.09	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
3433	122832	5.07	2.26	5.5 (5,5,6,6)	2.27 (2.40,2.40,2.14,2.14)
3511	119033	5.25	2.05	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3512	119345	5.25	2.05	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3513	120999	5.17	2.22	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
3521	119455	5.25	2.05	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3522	119585	5.25	2.05	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
3523	121331	5.17	2.22	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
3531	121256	5.17	2.22	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
3532	121477	5.17	2.22	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
3533	123132	5.10	2.39	5.5 (5,5,6,6)	2.27 (2.40,2.40,2.14,2.14)
3611	121743	5.15	2.07	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
3612	122055	5.15	2.07	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
3613	123710	5.07	2.24	5.5 (5,6,5,6)	2.27 (2.40,2.14,2.40,2.14)
3621	122282	5.15	2.07	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
3622	122412	5.15	2.07	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
3623	124158	5.07	2.24	5.5 (5,6,5,6)	2.27 (2.40,2.14,2.40,2.14)
3631	123966	5.07	2.24	5.5 (5,6,6,5)	2.27 (2.40,2.14,2.14,2.40)
3632	124187	5.07	2.24	5.5 (5,6,6,5)	2.27 (2.40,2.14,2.14,2.40)
3633	125842	5.00	2.41	5.8 (5,6,6,6)	2.20 (2.40,2.14,2.14,2.14)

The Summary of Dynamic Analysis Simulation (Continued)

Code	Total Cost 2011-2030 (Millions of Baht)	Cash Payment Efficiency		Average Number of Denominations (Period# 1/2/3/4)	Average Spacing Factor (Period# 1/2/3/4)
		Average No. of Minimum Tokens	Average No. of Efficient Schemes		
4111	115174	5.33	1.91	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
4112	115487	5.32	1.91	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
4113	117141	5.25	2.07	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
4121	115830	5.32	1.91	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
4122	115960	5.32	1.91	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
4123	117706	5.25	2.07	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
4131	117514	5.25	2.07	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
4132	117735	5.25	2.07	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
4133	119390	5.17	2.24	5.3 (5,4,6,6)	2.35 (2.40,2.72,2.14,2.14)
4211	115969	5.33	1.89	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
4212	116281	5.33	1.89	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
4213	117935	5.25	2.06	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
4221	116391	5.33	1.89	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
4222	116521	5.32	1.89	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
4223	118267	5.25	2.06	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
4231	118192	5.25	2.06	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
4232	118413	5.25	2.06	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
4233	120068	5.17	2.22	5.3 (5,4,6,6)	2.35 (2.40,2.72,2.14,2.14)
4311	118764	5.23	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4312	119077	5.23	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4313	120731	5.15	2.09	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
4321	119304	5.23	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4322	119434	5.22	1.92	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4323	121179	5.15	2.09	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
4331	120988	5.15	2.09	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
4332	121209	5.15	2.09	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
4333	122863	5.07	2.26	5.5 (5,5,6,6)	2.27 (2.40,2.40,2.14,2.14)
4411	118519	5.23	1.93	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4412	118832	5.23	1.93	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4413	120486	5.15	2.10	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
4421	119175	5.23	1.93	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4422	119305	5.23	1.93	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4423	121051	5.15	2.10	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
4431	120859	5.15	2.10	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
4432	121081	5.15	2.10	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
4433	122735	5.08	2.27	5.5 (5,5,6,6)	2.27 (2.40,2.40,2.14,2.14)
4511	119084	5.25	2.06	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4512	119396	5.25	2.06	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4513	121051	5.18	2.23	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
4521	119507	5.25	2.06	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4522	119636	5.25	2.06	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
4523	121382	5.17	2.23	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
4531	121308	5.18	2.23	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
4532	121529	5.17	2.23	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
4533	123183	5.10	2.40	5.5 (5,5,6,6)	2.27 (2.40,2.40,2.14,2.14)
4611	121794	5.15	2.08	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
4612	122107	5.15	2.08	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
4613	123761	5.08	2.25	5.5 (5,6,5,6)	2.27 (2.40,2.14,2.40,2.14)
4621	122334	5.15	2.08	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
4622	122463	5.15	2.08	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
4623	124209	5.07	2.25	5.5 (5,6,5,6)	2.27 (2.40,2.14,2.40,2.14)
4631	124018	5.08	2.25	5.5 (5,6,6,5)	2.27 (2.40,2.14,2.14,2.40)
4632	124239	5.07	2.25	5.5 (5,6,6,5)	2.27 (2.40,2.14,2.14,2.40)
4633	125893	5.00	2.42	5.8 (5,6,6,6)	2.20 (2.40,2.14,2.14,2.14)

The Summary of Dynamic Analysis Simulation (Continued)

Code	Total Cost 2011-2030 (Millions of Baht)	Cash Payment Efficiency		Average Number of Denominations (Period# 1/2/3/4)	Average Spacing Factor (Period# 1/2/3/4)
		Average No. of Minimum Tokens	Average No. of Efficient Schemes		
5111	115935	5.35	2.04	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
5112	116248	5.35	2.04	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
5113	117902	5.27	2.20	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
5121	116591	5.35	2.04	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
5122	116721	5.34	2.04	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
5123	118467	5.27	2.20	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
5131	118275	5.27	2.20	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
5132	118497	5.27	2.20	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
5133	120151	5.19	2.37	5.3 (5,4,6,6)	2.35 (2.40,2.72,2.14,2.14)
5211	116432	5.35	2.02	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
5212	116744	5.35	2.02	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
5213	118399	5.27	2.19	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
5221	116855	5.35	2.02	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
5222	116985	5.35	2.02	4.8 (5,4,5,5)	2.48 (2.40,2.72,2.40,2.40)
5223	118730	5.27	2.19	5.0 (5,4,5,6)	2.41 (2.40,2.72,2.40,2.14)
5231	118656	5.27	2.19	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
5232	118877	5.27	2.19	5.0 (5,4,6,5)	2.41 (2.40,2.72,2.14,2.40)
5233	120531	5.20	2.35	5.3 (5,4,6,6)	2.35 (2.40,2.72,2.14,2.14)
5311	119377	5.25	2.05	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5312	119689	5.25	2.05	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5313	121344	5.17	2.22	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
5321	119916	5.25	2.05	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5322	120046	5.25	2.05	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5323	121792	5.17	2.22	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
5331	121600	5.17	2.22	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
5332	121821	5.17	2.22	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
5333	123476	5.10	2.39	5.5 (5,5,6,6)	2.27 (2.40,2.40,2.14,2.14)
5411	119280	5.25	2.06	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5412	119593	5.25	2.06	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5413	121247	5.18	2.23	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
5421	119936	5.25	2.06	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5422	120066	5.25	2.06	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5423	121812	5.17	2.23	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
5431	121621	5.18	2.23	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
5432	121842	5.17	2.23	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
5433	123496	5.10	2.40	5.5 (5,5,6,6)	2.27 (2.40,2.40,2.14,2.14)
5511	119547	5.28	2.19	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5512	119860	5.27	2.19	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5513	121514	5.20	2.36	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
5521	119970	5.27	2.19	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5522	120100	5.27	2.19	5.0 (5,5,5,5)	2.40 (2.40,2.40,2.40,2.40)
5523	121846	5.20	2.36	5.3 (5,5,5,6)	2.33 (2.40,2.40,2.40,2.14)
5531	121771	5.20	2.36	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
5532	121992	5.20	2.36	5.3 (5,5,6,5)	2.33 (2.40,2.40,2.14,2.40)
5533	123646	5.12	2.53	5.5 (5,5,6,6)	2.27 (2.40,2.40,2.14,2.14)
5611	122406	5.18	2.21	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
5612	122719	5.17	2.21	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
5613	124373	5.10	2.38	5.5 (5,6,5,6)	2.27 (2.40,2.14,2.40,2.14)
5621	122946	5.17	2.21	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
5622	123076	5.17	2.21	5.3 (5,6,5,5)	2.33 (2.40,2.14,2.40,2.40)
5623	124821	5.10	2.38	5.5 (5,6,5,6)	2.27 (2.40,2.14,2.40,2.14)
5631	124630	5.10	2.38	5.5 (5,6,6,5)	2.27 (2.40,2.14,2.14,2.40)
5632	124851	5.10	2.38	5.5 (5,6,6,5)	2.27 (2.40,2.14,2.14,2.40)
5633	126505	5.02	2.55	5.8 (5,6,6,6)	2.20 (2.40,2.14,2.14,2.14)

The Summary of Dynamic Analysis Simulation (Continued)

Code	Total Cost 2011-2030 (Millions of Baht)	Cash Payment Efficiency		Average Number of Denominations (Period# 1/2/3/4)	Average Spacing Factor (Period# 1/2/3/4)
		Average No. of Minimum Tokens	Average No. of Efficient Schemes		
6111	119258	5.25	2.06	5.0 (6,4,5,5)	2.41 (2.14,2.72,2.40,2.40)
6112	119571	5.25	2.06	5.0 (6,4,5,5)	2.41 (2.14,2.72,2.40,2.40)
6113	121225	5.17	2.22	5.3 (6,4,5,6)	2.35 (2.14,2.72,2.40,2.14)
6121	119914	5.25	2.06	5.0 (6,4,5,5)	2.41 (2.14,2.72,2.40,2.40)
6122	120044	5.24	2.06	5.0 (6,4,5,5)	2.41 (2.14,2.72,2.40,2.40)
6123	121790	5.17	2.22	5.3 (6,4,5,6)	2.35 (2.14,2.72,2.40,2.14)
6131	121598	5.17	2.22	5.3 (6,4,6,5)	2.35 (2.14,2.72,2.14,2.40)
6132	121819	5.17	2.22	5.3 (6,4,6,5)	2.35 (2.14,2.72,2.14,2.40)
6133	123474	5.09	2.39	5.5 (6,4,6,6)	2.28 (2.14,2.72,2.14,2.14)
6211	119904	5.25	2.04	5.0 (6,4,5,5)	2.41 (2.14,2.72,2.40,2.40)
6212	120216	5.25	2.04	5.0 (6,4,5,5)	2.41 (2.14,2.72,2.40,2.40)
6213	121871	5.17	2.21	5.3 (6,4,5,6)	2.35 (2.14,2.72,2.40,2.14)
6221	120327	5.25	2.04	5.0 (6,4,5,5)	2.41 (2.14,2.72,2.40,2.40)
6222	120456	5.25	2.04	5.0 (6,4,5,5)	2.41 (2.14,2.72,2.40,2.40)
6223	122202	5.17	2.21	5.3 (6,4,5,6)	2.35 (2.14,2.72,2.40,2.14)
6231	122127	5.17	2.21	5.3 (6,4,6,5)	2.35 (2.14,2.72,2.14,2.40)
6232	122348	5.17	2.21	5.3 (6,4,6,5)	2.35 (2.14,2.72,2.14,2.40)
6233	124003	5.10	2.37	5.5 (6,4,6,6)	2.28 (2.14,2.72,2.14,2.14)
6311	122700	5.15	2.07	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6312	123012	5.15	2.07	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6313	124666	5.07	2.24	5.5 (6,5,5,6)	2.27 (2.14,2.40,2.40,2.14)
6321	123239	5.15	2.07	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6322	123369	5.15	2.07	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6323	125115	5.07	2.24	5.5 (6,5,5,6)	2.27 (2.14,2.40,2.40,2.14)
6331	124923	5.07	2.24	5.5 (6,5,6,5)	2.27 (2.14,2.40,2.14,2.40)
6332	125144	5.07	2.24	5.5 (6,5,6,5)	2.27 (2.14,2.40,2.14,2.40)
6333	126799	5.00	2.41	5.8 (6,5,6,6)	2.20 (2.14,2.40,2.14,2.14)
6411	122603	5.15	2.08	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6412	122916	5.15	2.08	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6413	124570	5.08	2.25	5.5 (6,5,5,6)	2.27 (2.14,2.40,2.40,2.14)
6421	123259	5.15	2.08	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6422	123389	5.15	2.08	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6423	125135	5.07	2.25	5.5 (6,5,5,6)	2.27 (2.14,2.40,2.40,2.14)
6431	124943	5.08	2.25	5.5 (6,5,6,5)	2.27 (2.14,2.40,2.14,2.40)
6432	125165	5.07	2.25	5.5 (6,5,6,5)	2.27 (2.14,2.40,2.14,2.40)
6433	126819	5.00	2.42	5.8 (6,5,6,6)	2.20 (2.14,2.40,2.14,2.14)
6511	123019	5.18	2.21	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6512	123332	5.17	2.21	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6513	124986	5.10	2.38	5.5 (6,5,5,6)	2.27 (2.14,2.40,2.40,2.14)
6521	123442	5.17	2.21	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6522	123572	5.17	2.21	5.3 (6,5,5,5)	2.33 (2.14,2.40,2.40,2.40)
6523	125317	5.10	2.38	5.5 (6,5,5,6)	2.27 (2.14,2.40,2.40,2.14)
6531	125243	5.10	2.38	5.5 (6,5,6,5)	2.27 (2.14,2.40,2.14,2.40)
6532	125464	5.10	2.38	5.5 (6,5,6,5)	2.27 (2.14,2.40,2.14,2.40)
6533	127118	5.02	2.55	5.8 (6,5,6,6)	2.20 (2.14,2.40,2.14,2.14)
6611	125729	5.08	2.23	5.5 (6,6,5,5)	2.27 (2.14,2.14,2.40,2.40)
6612	126042	5.07	2.23	5.5 (6,6,5,5)	2.27 (2.14,2.14,2.40,2.40)
6613	127696	5.00	2.40	5.8 (6,6,5,6)	2.20 (2.14,2.14,2.40,2.14)
6621	126269	5.07	2.23	5.5 (6,6,5,5)	2.27 (2.14,2.14,2.40,2.40)
6622	126399	5.07	2.23	5.5 (6,6,5,5)	2.27 (2.14,2.14,2.40,2.40)
6623	128144	5.00	2.40	5.8 (6,6,5,6)	2.20 (2.14,2.14,2.40,2.14)
6631	127953	5.00	2.40	5.8 (6,6,6,5)	2.20 (2.14,2.14,2.14,2.40)
6632	128174	5.00	2.40	5.8 (6,6,6,5)	2.20 (2.14,2.14,2.14,2.40)
6633	129828	4.92	2.57	6.0 (6,6,6,6)	2.14 (2.14,2.14,2.14,2.14)

BIOGRAPHY

NAME	Pat Pattanarangsun
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